Rigid transformations
Student Activity Sheet 3; use with Exploring “Reflections and rotations”

1. In the diagram, $\triangle A''B''C''$ is a composition of two reflections across lines $m_1$ and $m_2$. Compare how $\triangle ABC$ and $\triangle A''B''C''$ are oriented with respect to the intersecting lines. How can you tell from the images that $\triangle A''B''C''$ is not a translation of $\triangle ABC$? [EX2, page 2]

![Diagram of reflections]

In a translation, each point in a figure moves the same distance in the same direction. So, the line segments connecting corresponding points would have to be the same length and parallel. In this case, the line segments connecting the corresponding vertices of $\triangle ABC$ and $\triangle A''B''C''$ are not parallel or congruent. So, this transformation cannot be a translation.

2. Fill in the blanks to complete the following statements. [EX2, page 4]

A rotation of a point about a fixed point is a composite of two reflections of the point across intersecting lines. The point of intersection of the lines is the center of rotation.

3. Compare the orientation of $\triangle ABC$ and $\triangle A''B''C''$. Do you think your observation is true for all rotations? [EX2, page 4]

On both triangles, the vertices go in a counterclockwise direction, from $A$ to $B$ to $C$. Therefore, they have the same orientation. In general, rotations preserve orientation.
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4. Use the diagram to write another definition for rotation. [EX2, page 6]

![Diagram showing a rotation about a point P maps A to A'' so that ∠APA'' is the angle of rotation and PA = PA''].

A rotation about a point P maps A to A'' so that ∠APA'' is the angle of rotation and PA = PA''.

Use the terms provided to fill in the blanks in the statements below. [EX2, page 7]

<table>
<thead>
<tr>
<th>term</th>
<th>4x</th>
<th>rotation</th>
<th>transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>perpendicular bisector</td>
<td>reflection</td>
<td>translation</td>
<td>x</td>
</tr>
</tbody>
</table>

5. A rigid **transformation** preserves distance, angle measure, and area.

6. Point P' is a reflection of P across line m if and only if m is the **perpendicular bisector** of PP'.

7. A **translation** is a composition of two reflections over two parallel lines. If the distance between the lines is 2x, then the distance between the image and the pre-image is 4x.

8. A **rotation** is a composition of two reflections over two intersecting lines. If the measure of the angle of rotation is 2x, then the measure of the acute angle of the intersecting lines is x.
9. **REINFORCE** In the diagram, assume the measure of the acute angle between \( m_1 \) and \( m_2 \) is 80°, and \( m\angle APA'' = (5x - 30)° \). Solve for \( x \).

\[
\begin{align*}
m\angle APA'' & \text{ is twice the measure of the acute angle formed by lines } m_1 \text{ and } m_2. \\
5x - 30 & = 2(80) \\
5x - 30 & = 160 \\
5x & = 190 \\
x & = 38
\end{align*}
\]

10. **REINFORCE** In the diagram, assume \( PA = (6x + 4) \) inches and \( PA'' = (7x + 1) \) inches. Solve for \( x \).

Since \( A'' \) is a rotation of \( A \) about \( P \), \( PA = PA'' \).

\[
\begin{align*}
6x + 4 & = 7x + 1 \\
3 & = x
\end{align*}
\]