Orange Public Schools

Office of Curriculum & Instruction
2019-2020 Mathematics Curriculum Guide

8th Grade Mathematics

Illustrative Mathematics – Unit 8: Pythagorean Theorem and Irrational Numbers

May 6, 2020 – June 3, 2020

Board Approved: 1.14.2020

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From the New Jersey Student Learning Standards:

In **Grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Yearlong Pacing Guide
Grade 8

Grade	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
5	Unit 1 5.NBT	Unit 2 5.NBT		Unit 3 5.NF	Unit 4 5.NF		Unit 5 5.MD		Unit 6 5.OA & 5.G	
6	Unit 1 6.G	Unit 2 6.RP	Unit 3 6.RP	Unit 4 6.NS		Unit 5 6.NS	Unit 6 6.EE	Unit 7 6.NS	Unit 8 6.SP	
7	Unit 1 7.G	Unit 2 7.RP	Unit 3 7.G	Unit 4 7.RP	Unit 5 7.NS	Unit 6 7.EE		Unit 7 7.G	Unit 8 7.SP	
8	**Unit 1 8.G**	**Unit 2 8.G**	**Unit 3 8.EE**	**Unit 4 8.EE**		**Unit 5 8.F**	**Unit 6 8.SP**	**Unit 7 8.EE**	**Unit 8 8.G**	

Unit 1	**Geometry:** Rigid Transformation & Congruence	Unit 2	**Geometry:** Dilations, Similarity, and Introducing Slope	Unit 3	**Expressions & Equations:** Linear Relationships	Unit 4	**Expressions & Equations:** Linear Equations & Linear Systems
Unit 5	**Functions:** Functions and Volume	Unit 6	**Statistics & Probability:** Associations in Data	Unit 7	**Expressions & Equations:** Exponents and Scientific Notation	Unit 8	**Geometry:** Pythagorean Theorem and Irrational Numbers

2019-2020 Grade 8 (iM)							
Quarter 1		**Quarter 2**		**Quarter 3**		**Quarter 4**	
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
8.G.1(M) 8.G.2(M) 8.G.5(M)	8.G.4(M) 8.G.3(M) 8.EE.6(M)	8.EE.5(M) 8.F.4(S) 8.EE.8(M)	8.EE.7(M) 8.EE.8(M)	8.F.1(M) 8.F.2(M) 8.F.3(M) 8.F.4(S) 8.F.5(S) 8.G.9(A)	8.SP.1(S) 8.SP.2(S) 8.SP.3(S) 8.SP.4(S)	8.EE.1(M) 8.EE.3(M) 8.EE.4(M)	8.NS.2(S) 8.EE.2(M) 8.G.6(M) 8.G.7(M) 8.G.8(M) 8.NS.1(S)
20 Days	15 Days	17 Days	18 Days	25 Days	13 Days	18 Days	17 Days
Oct. 8	Nov. 4	Dec. 6	Jan. 15	Mar. 4	Mar. 27	May 5	Jun. 3

Major Work Supporting Content Additional Content

Table of Contents

References

"Illustrative Mathematics" *Open Up Resources.* 2018 <https://auth.openupresources.org/register/complete>

I. Unit Overview

The unit begins by foreshadowing algebraic and geometric aspects of the Pythagorean Theorem and strategies for proving it. Students are shown three squares and asked to compare the area of the largest square with the sum of the areas of the other two squares. The comparison can be done by counting grid squares and comparing the counts—when the three squares are on a grid with their sides on grid lines and vertices on intersections of grid lines—using the understanding of area measurement established in grade 3. The comparison can also be done by showing that there is a shape that can be decomposed and rearranged to form the largest square or the two smallest squares. Students are provided with opportunities to use and discuss both strategies.

In the second section, students work with figures shown on grids, using the grids to estimate lengths and areas in terms of grid units, e.g., estimating the side lengths of a square, squaring their estimates, and comparing them with estimates made by counting grid squares. The term "square root" is introduced as a way to describe the relationship between the side length and area of a square (measured in units and square units, respectively), along with the notation √. Students continue to work with side lengths and areas of squares. They learn and use definitions for "rational number" and "irrational number." They plot rational numbers and square roots on the number line. They use the meaning of "square root," understanding that if a given number p is the square root of n, then x^2=n. Students learn (without proof) that √2 is irrational. They understand two proofs of the Pythagorean Theorem—an algebraic proof that involves manipulation of two expressions for the same area and a geometric proof that involves decomposing and rearranging two squares. They use the Pythagorean Theorem in two and three dimensions, e.g., to determine lengths of diagonals of rectangles and right rectangular prisms and to estimate distances between points in the coordinate plane.

In the third section, students work with edge lengths and volumes of cubes and other rectangular prisms. (In this grade, all prisms are right prisms.) They are introduced to the term "cube root" and the notation √3. They plot square and cube roots on the number line, using the meanings of "square root" and "cube root," e.g., understanding that if a given number x is the square root of n and n is between m and p, then x^2 is between m and p and that x is between √m and √p

In the fourth and last section, students work with decimal representations of rational numbers and decimal approximations of irrational numbers. In grade 7, they used long division in order to write fractions as decimals and learned that such decimals either repeat or terminate. They build on their understanding of decimals to make successive decimal approximations of √2 and π which they plot on number lines.

Essential Questions

- What strategies can you use to compare and order rational and irrational numbers on a number line?
- How can you distinguish between rational and irrational numbers?
- How are the lengths of the sides of a right triangle related?
- How does the Pythagorean Theorem relate the side lengths of a right triangle?
- How can you determine if a triangle is a right triangle?
- What kind of problems can be solved using the Pythagorean Theorem?
- How can you use the Pythagorean Theorem to find the distance between two points?

Enduring Understanding

- A rational number is a number (value) within the real number system that can be expressed as a fraction, a/b where *a* and *b* are integers and b ≠ 0). Rational numbers consist of fractions that either terminate or repeat.
- An irrational number is a number (value) within the real number system that cannot be expressed as a fraction, a/b, where *a* and *b* are integers. An irrational number is a decimal that never terminates or repeats.
- Many real-world problem situations can be represented with a mathematical model, but that model may not represent a real world, solution exactly.
- The Pythagorean Theorem can be used to determine if a triangle is a right triangle and to find the missing side length of a triangle.
- If a triangle has a length such that, the triangle is a right triangle.
- The Pythagorean Theorem and its converse can be used to solve real-world problems that involve right triangles. Both can be used to determine the unknown leg lengths of a right triangle, or to identify or verify whether a triangle is a right triangle.
- The Pythagorean Theorem can be used to find the distance between any two points on a coordinate plane by drawing a line to connect the points as the hypotenuse of the right triangle where the leg are the horizontal and vertical distances.

II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSLS)	Estimated Time (Blocks)
Unit 8 Pre-Unit Assessment *Optional*	6.G.A.3, 6.EE.A.1, 6.NS.C.6.c, 4.NF.C, 5.NBT.A.3, 7.NS.A.2.d, 6.G.A.1, 8.EE.A	½
Lesson 1: The Areas of Squares and Their Side Lengths	8.NS.A.2	1
Lesson 2: Side Lengths and Areas	8.EE.A.2, 8.F.B, 8.NS.A	1
Lesson 3: Rational and Irrational Numbers	8.EE.A.2, 8.NS.A	1
Lesson 4: Square Roots on the Number Line	8.EE.A.2, 8.NS.A.2	1
Lesson 5: Reasoning About Square Roots	8.EE.A.2, 8.NS.A.2	1
Lesson 6: Finding Side Lengths of Triangles	8.G.B, 8.G.B.7	1
Lesson 7: A Proof of the Pythagorean Theorem	8.G.B, 8.G.B.6, 8.G.B.7	1
Lesson 8: Finding Unknown Side Lengths	8.G.B.7	1
Lesson 9: The Converse	8.G.B, 8.G.B.6	1
Lesson 10: Applications of the Pythagorean Theorem	8.EE.A.2, 8.B.7, 8.NS.A	1
Lesson 11: Finding Distance	8.G.B.8	1
Lesson 12: Edge Lengths and Volumes	8.EE.A.2	1
Lesson 13: Cube Roots	8.EE.A.2, 8.NS.A.2	1
Lesson 14: Decimal Representations of Rational Numbers	8.EE.A, 8.NS.A, 8.NS.A.2	1
Lesson 15: Infinite Decimal Expansions ***(Project Based Learning)***	8.NS.A.1	1
Performance Task	8.G.B.8	½
Unit 8 End of Unit Assessment *Optional*	8.EE.A.2, 8.G.B, 8.NS.A.1, 8.EE.A.2, 8.NS.A.2, 8.G.B.8, 8.NS.A.2, 8.G.B.7, 8.G.B.9	**1**
Total Time		**17 Blocks**
Grade 8 Interim Assessment 4	8.EE.A.1, 8.EE.A.3, 8.G.B.6, 8.G.B.7, 8.G.B.8	**1**

Major Work Supporting Content Additional Content

III. Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2)*.

MAY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2)*.

JUNE

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	**2**	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**	**13**
14	**15**	**16**	**17**	**18**	**19**	**20**
21	**22**	**23**	**24**	**25**	**26**	**27**
28	**29**	**30**				

IV. NJSLA Assessment Evidence Statements

Type I Type II Type III

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number.	i) Tasks do not have a context. ii) An equal number of tasks require students to write a fraction a/b as a repeating decimal, or write a repeating decimal as a fraction. iii) For tasks that involve writing a repeating decimal as a fraction, the given decimal should include no more than two repeating decimals without non-repeating digits after the decimal point (i.e. 2.16666…, 0.23232323…).	MP. 7 MP. 8	No
8.NS.2	Use rational approximations of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g. π^2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.	i) Tasks do not have a context.	MP. 5 MP. 7 MP. 8	No
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form x^2=p and x^3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	i) Tasks may or may not have a context. ii) Students are not required to simplify expressions such as √8 to 2√2. Students are required to express the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100; and the cube roots of 1, 8, 27,	MP. 7	No

		and 64.		
8.G.1a	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.	i) Tasks do not have a context	MP. 3 MP. 5 MP. 8	No
8.G.1b	Verify experimentally the properties of rotations, reflections, and translations: b. Angles are taken to angles of the same measure.	i) Tasks do not have a context	MP. 3 MP. 5 MP. 8	No
8.G.7-1	Apply the Pythagorean Theorem in a simple planar case.	i) Tasks have "thin context" 2 or no context. ii) An equal number of tasks require the answer to be given as a whole number or as an irrational number written to approximately three decimal places.		Yes
8.G.7-2	Apply the Pythagorean Theorem in a simple three-dimensional case.	i) Tasks have "thin context" 2 or no context. ii) An equal number of tasks require the answer to be given as a whole number or as an irrational number written to approximately three decimal places.		Yes
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.			Yes
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.		MP. 1 MP. 5	Yes
8.C.3.1	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.F.3-2	i) Tasks require students to justify whether a given function is linear or nonlinear.	MP. 3 MP. 6	Yes
8.C.3.2	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4		MP.3 MP.5 MP.6	Yes

8.C.3.3	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.5		MP.3 MP.5 MP.6	Yes
8.C.5.2	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4		MP.2 MP.3 MP.5	Yes
8.C.5.3	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.G.B	i) Some of tasks require students to use the converse of the Pythagorean Theorem.	MP.2 MP.3 MP.5	Yes

V. Differentiated Instruction

Supporting English Language Learners

The purpose of this document is to nudge the field forward by offering support to the next generation of mathematics learners and by challenging persistent assumptions about how to support and develop students' disciplinary language. The goal is to provide guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense making. UL/SCALE provides a framework for organizing strategies and special considerations to support students in learning mathematics practices, content, and language. The framework is intended to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while the framework can and should be used to support all students learning mathematics, it is particularly well-suited to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English.

For more information, click the link below:

Supporting ELL Learners

Supporting Students with Disabilities

The philosophical stance that guided the creation of these materials is the belief that with proper structures, accommodations, and supports, all children can learn mathematics. Lessons are designed to maximize access for all students and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

For more information, click the link below:

Supporting Students with Disabilities

VI. Vocabulary

Cube Root:	The cube root of a number x, written 3√x, is the number whose cube is x. For example, 3√8=2 because 2^3=8. The cube root of 0 is 0.
Hypotenuse:	In a right triangle, the side opposite the right angle is called the hypotenuse.
Irrational Number:	The square root of a positive number x, written x—√, is the positive number whose square is x. For example, 4—√=2 because 22=4 and 2 is positive. The square root of 0 is 0.
Legs:	In a right triangle, the sides that are adjacent to the right angle are called the legs.
Rational Number:	A rational number is a fraction or the opposite of a fraction. Remember that a fraction is a point on the number line that you get by dividing the unit interval into b equal parts and finding the point that is a of them from 0. We can always write a fraction in the form *a/b* where *a* and *b* are whole numbers, with b not equal to 0, but there are other ways to write them. For example, 0.7 is a fraction because it is the point on the number line you get by dividing the unit interval into 10 equal parts and finding the point that is 7 of those parts away from 0. We can also write this number as 7/10.
Pythagorean Theorem:	The Pythagorean Theorem is a name for the property of all right triangles that the square of the hypotenuse is equal to the sum of the squares of the legs. It is often expressed as a^2+b^2=c^2.
Square Root:	The square root of a positive number x, written √x, is the positive number whose square is x. For example, √4=2 because 2^2=4 and 2 is positive. The square root of 0 is 0.

VII. Assessment Framework

Unit 8 Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Pre-Unit Diagnostic Assessment (Beginning of Unit – Optional) *Illustrative Mathematics*	6.G.A.3, 6.EE.A.1, 6.NS.C.6.c, 4.NF.C, 5.NBT.A.3, 7.NS.A.2.d, 6.G.A.1, 8.EE.A	½ Block	Individual	Yes (No Weight)
End-of-Unit Assessment (End of Unit – Optional) *Illustrative Mathematics*	8.EE.A.2, 8.G.B, 8.NS.A.1, 8.EE.A.2, 8.NS.A.2, 8.G.B.8, 8.NS.A.2, 8.G.B.7, 8.G.B.9	1 Block	Individual	Yes
Grade 8 Interim Assessment 4 (Mid June) *iReady Standards Mastery*	8.EE.A.1, 8.EE.A.3, 8.G.B.6, 8.G.B.7, 8.G.B.8	1 Block	Individual	Yes

Unit 8 Performance Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 8 Performance Task 1 (Early June) *Finding the Distance Between Points*	8.G.B.8	½ Block	Individual	Yes; Rubric
Unit 8 Performance Task Option 1 (Optional) *What's Your Favorite Subject?*	8.G.B.8	Teacher Discretion	Teacher Discretion	Yes, if administered
Extended Constructed Response (ECR)* (click here for access)	Dependent on unit of study & month of administration	Up to 30 minutes	Individual	Yes; Rubric

* Use the following links to access ECR protocol and district assessment scoring documents:

- *Assessment & Data in Mathematics Bulletin*
- *Extended Constructed Response Protocol*

Name _____ Block _____ Date _____

Finding the Distance Between Points (8.G.B.8)

Task:

a. Plot the points (5,3), (-1,1), and (2,-3) in the coordinate plane and find the lengths of the three segments connecting the points.
b. Find the distance between (5,9) and (-4,2) without plotting the points.
c. If (u,v) and (s,t) are two distinct points in the plane, what is the distance between them? Explain how you know.
d. Does your answer to (c) agree with your calculations in parts (a) and (b)? Explain.

8th Grade Finding the Distance Between Points Name: _____ Date: _____

NJSLS: 8.G.B.8 Type: _____ Teacher: _____

SOLUTION

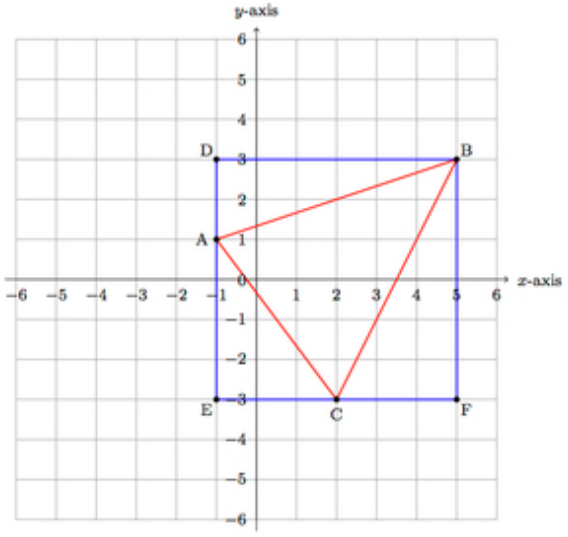
a. Student accurately plots and labels the three points and the segments joining them.
b. Student correctly implements the Pythagorean Theorem to find the distance between the two points.
c. Student applies the pattern in part (b), the distance formula, to describe the distance between (u,v) to (s,t).

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

8th Grade Finding the Distance Between Points – Scoring Guide

Answer

The three points and the segments joining them are plotted and labeled below:



To calculate the distance between two of the points, A and B for example, we can use the Pythagorean Theorem provided we can find a right triangle which has $\overline{AB}$ as one side. The vertical and horizontal grid lines are perpendicular to one another so we can make a right angle by choosing one vertical grid line segment and one horizontal grid line segment as the legs of our triangle. This is also pictured above as $\triangle ADB$ is a right triangle with right angle D. We can apply the Pythagorean theorem to $\triangle ADB$ to find

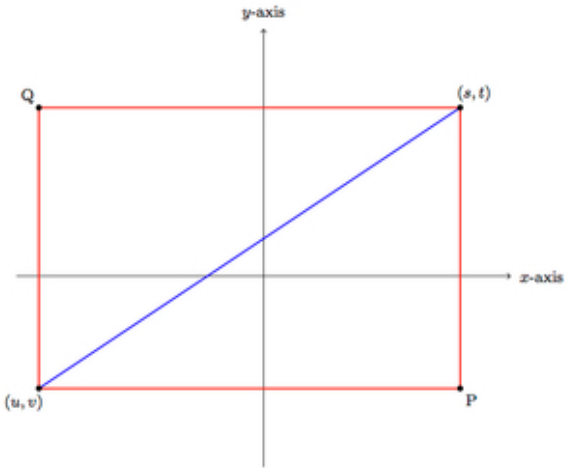
$$|AB|^2 = |AD|^2 + |BD|^2.$$

We know $|AD| = 2$ units and $|BD| = 6$ units by counting squares on the coordinate grid. So this means $|AD| = \sqrt{40}$ units. Applying the same technique to $\triangle AEC$ with right angle E we find $|AC| = \sqrt{25} = 5$ units. Using $\triangle CFB$ we find that $|CB| = \sqrt{45}$ units.

b. Building on the work in part (a), we can look for a pattern in the calculations. In each case we take a square root and we are taking the square root of the square of the horizontal change in the point coordinates added to the square of the vertical change in the point coordinates. Applying this idea to (5,9) and (-4,2), the horizontal change in these coordinates is $5 - (-4) = 9$ while the vertical change is $9 - 2 = 7$. The squares of these numbers are $9^2 = 81$ and $7^2 = 49$. Then the square root of the sum of these squares is $\sqrt{81 + 49} = \sqrt{130}$. So the segment joining $(5, 9)$ to $(-4, 2)$ has length $\sqrt{130}$ units.

c.
Looking at part (b), we need to find the horizontal and vertical distance covered to go from (u, v) to (s, t). Moving from u to s requires a horizontal displacement of $s - u$ units while going from v to t is a vertical displacement of $t - v$ units. Applying the pattern in part (b), the distance from (u, v) to (s, t) is $\sqrt{(s-u)^2 + (t-v)^2}$.

We can verify this formula and pattern by drawing a representative picture as in part (a), though we have not labeled the numbers on the axes because we do not know the exact coordinates of the points:



There are many other possible pictures as (u, v) or (s, t) could lie on one of the coordinate axes or could be in different quadrants. The Pythagorean relationship, however, holds regardless of where we translate the image.

In order to find the distance from (u, v) to (s, t) we can make these points two vertices of a rectangle, with horizontal and vertical sides, as drawn above. We let $A = (s, t)$ and $B = (u, v)$ in the following calculations. Since side $\overline{BQ}$ is vertical, this means that the x-coordinate of Q is the same as that of $B = (u, v)$, namely u. Similarly, the y-coordinate of Q is the same as the y-coordinate of $A = (s, t)$, namely t. So $Q = (u, t)$. Leg $\overline{BQ}$ has length $|t - v|$ while leg QA has length $|s - u|$. Applying the Pythagorean theorem we find that the distance from (u, v) to (s, t) is $\sqrt{|s - u|^2 + |t - v|^2}$. This is the same as what we found above: the square of any number is non-negative so $|s - u|^2 = (s - u)^2$ and $|t - v|^2 = (t - v)^2$.

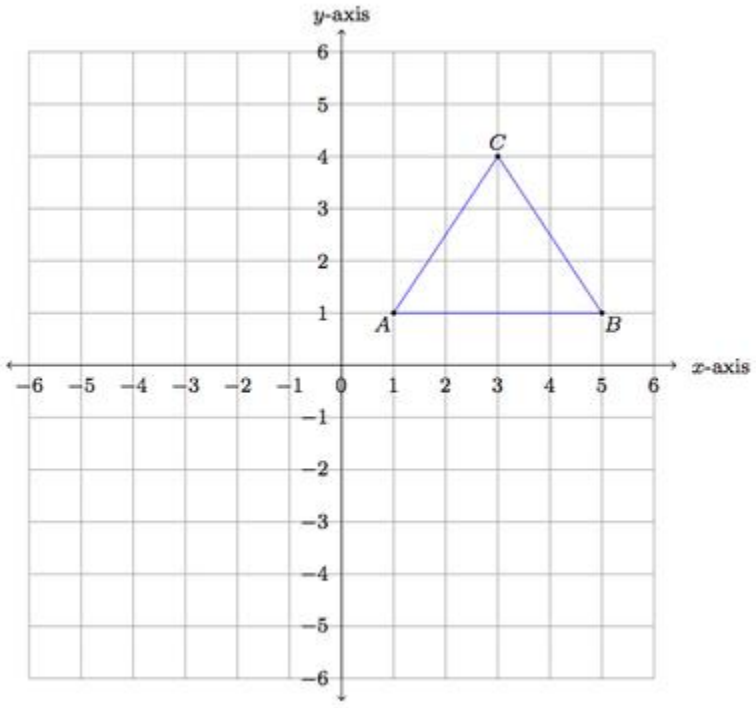
d. Here we apply the formula found in part (c) with $(u, v) = (5, 3)$ and $(s, t) = (-1, 1)$. We find that the distance between these points is $\sqrt{(-1-5)^2 + (1-3)^2} = \sqrt{40}$, agreeing with our calculations in part (a). The formula gives the same answers as we calculated above in all of the rest of the cases as well.

Name _____ Block _____ Date _____

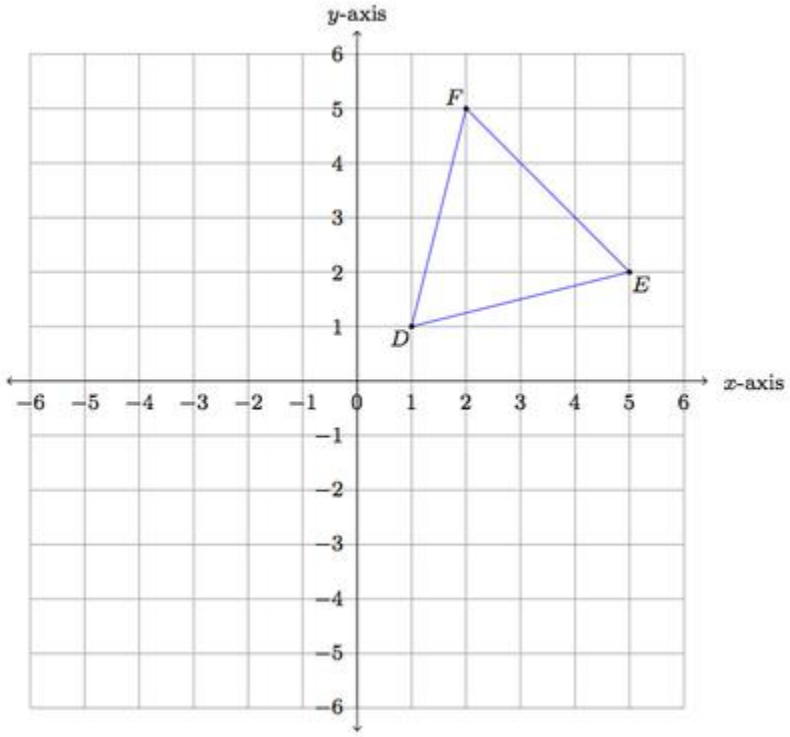
Finding Isosceles Triangles (8.G.B.8)

Mrs. Lu has asked students in her class to find isosceles triangles whose vertices lie on a coordinate grid. For each student example below, explain why the triangle is isosceles.

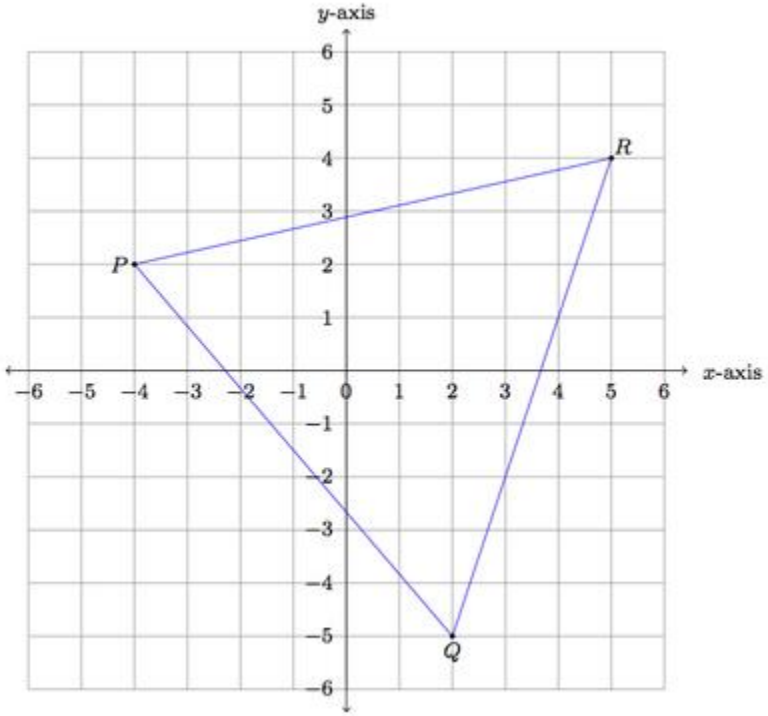
a. Jessica draws the following triangle:



b. Bruce's picture is here:



c. Yan draws this picture:



IX. Modifications

Special Education/ 504:	English Language Learners:
-Adhere to all modifications and health concerns stated in each IEP. -Give students a MENU options, allowing students to pick assignments from different levels based on difficulty. -Accommodate Instructional Strategies: reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), handouts, definition list with visuals, extended time -Allow students to demonstrate understanding of a problem by drawing the picture of the answer and then explaining the reasoning orally and/or writing , such as Read-Draw-Write -Provide breaks between tasks, use positive reinforcement, use proximity -Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum by using manipulatives -Implement supports for students with disabilities (click here) - Make use of strategies imbedded within lessons -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 17-18) - Strategies for students with 504 plans	- Use manipulatives to promote conceptual understanding and enhance vocabulary usage - Provide graphic representations, gestures, drawings, equations, realia, and pictures during all segments of instruction - During i-Ready lessons, click on “Español” to hear specific words in Spanish - Utilize graphic organizers which are concrete, pictorial ways of constructing knowledge and organizing information - Use sentence frames and questioning strategies so that students will explain their thinking/ process of how to solve word problems - Utilize program translations (if available) for L1/ L2 students - Reword questions in simpler language - Make use of the ELL Mathematical Language Routines (click here for additional information) -Scaffolding instruction for ELL Learners -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 16-17)
Gifted and Talented:	**Students at Risk for Failure:**
- Elevated contextual complexity - Inquiry based or open ended assignments and projects - More time to study concepts with greater depth - Promote the synthesis of concepts and making real world connections - Provide students with enrichment practice that are imbedded in the curriculum such as: ● Application / Conceptual Development ● Are you ready for more? - Provide opportunities for math competitions - Alternative instruction pathways available - Common Core Approach to Differentiate Instruction: Students with Disabilities (pg. 20)	- Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum - Modify Instructional Strategies, reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), inclusion of more visuals and manipulatives, Peer Support - Constant parental/ guardian contact - Provide academic contracts to students & guardians - Create an interactive notebook with samples, key vocabulary words, student goals/ objectives. - Plan to address students at risk in your learning tasks, instructions, and directions. Anticipate where the needs will be, then address them prior to lessons. -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 19)

21st Century Life and Career Skills:

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

https://www.state.nj.us/education/cccs/2014/career/9.pdf

- **CRP1**. Act as a responsible and contributing citizen and employee.
- **CRP2**. Apply appropriate academic and technical skills.
- **CRP3**. Attend to personal health and financial well-being.
- **CRP4**. Communicate clearly and effectively and with reason.
- **CRP5**. Consider the environmental, social and economic impacts of decisions.
- **CRP6**. Demonstrate creativity and innovation.
- **CRP7**. Employ valid and reliable research strategies.
- **CRP8**. Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP9**. Model integrity, ethical leadership and effective management.
- **CRP10**. Plan education and career paths aligned to personal goals.
- **CRP11**. Use technology to enhance productivity.
- **CRP12**. Work productively in teams while using cultural global competence.

Students are given an opportunity to communicate with peers effectively, clearly, and with the use of technical language. They are encouraged to reason through experiences that promote critical thinking and emphasize the importance of perseverance. Students are exposed to various mediums of technology, such as digital learning, calculators, and educational websites.

Technology Standards:

All students will be prepared to meet the challenge of a dynamic global society in which they participate, contribute, achieve, and flourish through universal access to people, information, and ideas.
https://www.state.nj.us/education/cccs/2014/tech/

8.1 Educational Technology:

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

A. **Technology Operations and Concepts:** Students demonstrate a sound understanding of technology concepts, systems and operations.
B. **Creativity and Innovation:** Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
C. **Communication and Collaboration:** Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
D. **Digital Citizenship:** Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
E. **Research and Information Fluency:** Students apply digital tools to gather, evaluate, and use of information.
F. **Critical thinking, problem solving, and decision making:** Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

A. **The Nature of Technology: Creativity and Innovation-** Technology systems impact every aspect of the world in which we live.
B. **Technology and Society:** Knowledge and understanding of human, cultural, and societal values are fundamental when designing technological systems and products in the global society.
C. **Design:** The design process is a systematic approach to solving problems.
D. **Abilities in a Technological World:** The designed world in a product of a design process that provides the means to convert resources into products and systems.
E. **Computational Thinking: Programming-** Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

Interdisciplinary Connections:	
English Language Arts:	
L.8.3	Use knowledge of language and its conventions when writing, speaking, reading, or listening.
SL.8.1	Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clearly.
W.8.1	Write arguments to support claims with clear reasons and relevant evidence.

X. Core Instruction & Supplemental Resources

Core Instruction

ILLUSTRATIVE MATHEMATICS V. 2019
(OPEN UP RESOURCES)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
6	• **Teacher Edition: Unit 1-9** • Online Course Guide	• Student Workbook Set: Unit 1-9 • Online Student Access (Digital Applets)
7	• **Teacher Edition: Unit 1-9** • Online Course Guide	• Student Workbook Set: Unit 1-9 • Online Student Access (Digital Applets)
8	• **Teacher Edition: Unit 1-9** • Online Course Guide	• Student Workbook Set: Unit 1-9 • Online Student Access (Digital Applets)

5 Practices for Orchestrating Productive Mathematics Discussions

Anticipate

Consider how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that you would like the students to learn.

- Solve the problem yourself first. If possible work with colleagues.
- Ask yourself the following questions:
  - What strategies have students used in the past?
  - What representations are students most likely to use?
  - What incorrect or unproductive strategies are students likely to try?
  - What things might get in the way of students being able to engage with the problem? How can you remove those barriers?
  - What questions will you ask those who struggle?

Monitor

Pay close attention to students' mathematical thinking and solution strategies as they work on the task.

- Create a list of strategies the students may produce.
- Circulate the room. Watch and listen to students as they work.
- If any students use strategies you anticipated, write their name or group number on your list.
- Ask questions that will help students make their thinking visible.
- Ask questions that will help students clarify their thinking.
- Press students to consider aspects of the task to which they need to attend.

Select

Select particular students to share their work with the rest of the class to get specific mathematics into the open for discussion. The selection of particular students and their solutions is guided by the previously anticipated strategies and your assessment of how each approach will contribute to that goal.

- Based on the previously anticipated strategies and the mathematical goal of the activity, decide which student strategies to highlight.
- Select students who will share their work with the class.

Sequence

Make purposeful choices about the order in which students' work is shared to maximize the chances of achieving the mathematical goals for the discussion.

- Based on the mathematical goal, decide on the purpose for the sequence of work. For example: least efficient to most efficient, concrete to abstract, misconceptions to conceptions, or building representations.
- Decide in which order students will present their work.

Connect

Help students draw connections between their solutions and other students' solutions as well as the key mathematical ideas in the lesson. Help students to make judgments about the consequences of different approaches for the range of problems that can be solved, one's likely accuracy and efficiency in solving them, and the kinds of mathematical patterns that can be most easily discerned. Know where you want the discussion to "land" and make choices that are likely to get you there. If necessary, you may have to demonstrate an approach that students didn't come up with themselves.

- As students share, ask questions to elicit and clarify student thinking.
- After each student shares, ask questions to connect it to previously shared work or ask a student to summarize what another student said in their own words.
- Ask students to compare and contrast strategies or representations during the discussion.
- If students did not come up with an approach that you need them to see in order for the discussion to "land," demonstrate this approach and connect it to the work that students did.

IDEAL MATH BLOCK

Whole Group Instruction	55min	INSTRUCTION (Grades 3 – 8) Daily Routine: Mathematical Content or Language Routine (7 – 10 min) Anchor Task: Anticipate, Monitor, Select, Sequence, Connect Tech Integration: Digital applets embedded within lessons designed to enhance student learning Collaborative Work* Guided Learning/Guided Practice Independent Work (Demonstration of Student Thinking) Additional Activities / Let's Practice		
Rotation Stations (Student Notebooks & Chromebooks Needed)	1-2X 30 min	STATION 1: Focus on current Grade Level Content STUDENT EXPLORATION* Independent or groups of 2-3 Emphasis on MP's 3, 6 (Reasoning and Precision) And MP's 1 & 4 (Problem Solving and Application) TOOLS/RESOURCES Practice Problems Extra Practice/Enrichment Are you ready for more? Put Your Thinking Cap On	STATION 2: Focus on Student Needs TECH STATION Independent TECH INTEGRATION iReady - *i-Ready* delivers online lessons driven by student data to provide tailored instruction that meets students where they are in their learning trajectory. Dreambox (ELL) – Adaptive online learning platform.	TEACHER STATION: Focus on Grade Level Content; heavily scaffolded to connect deficiencies TARGETED INSTRUCTION 4 – 5 Students TOOLS/ RESOURCES Homework Manipulatives Reteach Workbook Transition Guide *all students seen in 2 weeks
Closure	5 min	INSTRUCTION Exit Ticket (Demonstration of Student Thinking) TOOLS/RESOURCES Notebooks or Exit Ticket Slips		

* Promotes discourse and collaboration

Supplemental Resources

Achieve the Core

Tasks - https://achievethecore.org/category/416/mathematics-tasks

Coherence Map - https://achievethecore.org/page/1118/coherence-map

Embarc

https://embarc.online/

Engage NY

https://www.engageny.org/ccss-library/?f%5B0%5D=field_subject%253Aparents_all%3A13601

iReady Digital Platform

https://login.i-ready.com/

Illustrative Mathematics

Content Standard Tasks - https://tasks.illustrativemathematics.org/content-standards

Practice Standard Tasks - https://tasks.illustrativemathematics.org/practice-standards

Open Up Resources - https://access.openupresources.org/sign_in

iM Additional Resources - https://bit.ly/imshare

Khan Academy

https://www.khanacademy.org/math/illustrative-math

NJDOE Digital Item Library

https://nj.digitalitemlibrary.com/home?subject=Math

Ready Teacher Toolbox

https://teacher-toolbox.com/