8th Grade Mathematics

Linear Equations and Linear Systems Unit 4 Pacing Calendar – Illustrative Mathematics



ORANGE PUBLIC SCHOOLS OFFICE OF CURRICULUM AND INSTRUCTION OFFICE OF MATHEMATICS

Revised: 10/22/2019

From the New Jersey Student Learning Standards:

In **Grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m·A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Yearlong Pacing Guide Grade 8

Grade	SEP	0	СТ	N	VC	D	EC JA	N	FE	В	MA	R	A	R	Μ	AY	JUN
5	Unit 1 5.NBT		Un 5.N	it 2 IBT			Unit 3 5.NF		Un 5.1	it 4 NF		l	Jnit 5 5.MD			Unit 6 5.OA & 5.	G
6	Unit 1 6.G		Un 6.1	it 2 RP	Un 6.1	it 3 RP	Unit 4 6.NS			Unit 5 6.NS		Unit 6.E	E E	Unit 6.N	:7 S	Unit 6.S	: 8 P
7	Unit 1 7.G	Uni 7.F	it 2 RP	Un 7.	it 3 G		Unit 4 7.RP	Un 7.	it 5 NS	Ur 7	nit 6 '.EE			Unit 7 7.G		Unit 8 7.SP	
8	Unit 1 8.G	L	Un 8.	it 2 .G	Un 8.	it 3 EE	Unit 4 8.EE			Unit 5 8.F		Unit 8.S	t6 P	Unit 8.E	: 7 E	Unit 8 8.G	
	Unit 1	Geom Transi & Cor	ietry: I forma ngruen	Rigid tion Ice	Unit 2		Geometry: Dilations, Similarity, ar Introducing Slope	nd	Unit 3	t Ex Ec Ri	xpress quatic elation	sions ons: L nship	& inear s	Unit 4		Expression Equations Equations Linear Sys	ns & s: Linear ; & stems
	Unit 5	Funct Volum	ions: ions a ne	ind	Unit 6		Probability: Associations Data	in	Unit 7	E E E S N	xpress quatic xpone cientif lotatio	sions ons: ents a fic on	a nd	Unit 8		Pythagore Theorem Irrational Numbers	/: ean and

	2019-2020 Grade 8 (iM)										
Quarter 1		Quarter 2		Quar	rter 3	Quarter 4					
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8				
8.G.1(M) 8.G.2(M) 8.G.5(M)	8.G.4(M) <mark>8.G.3(M)</mark> 8.EE.6(M)	8.EE.5(M) 8.F.4(S) 8.EE.8(M)	8.EE.7(M) 8.EE.8(M)	8.F.1(M) 8.F.2(M) 8.F.3(M) 8.F.4(S) 8.F.5(S) 8.G.9(A)	8.SP.1(S) 8.SP.2(S) 8.SP.3(S) 8.SP.4(S)	8.EE.1(M) 8.EE.3(M) 8.EE.4(M)	8.NS.2(S) 8.EE.2(M) 8.G.6(M) 8.G.7(M) 8.G.8(M) 8.NS.1(S)				
20 Days	15 Days	17 Days	18 Days	25 Days	13 Days	18 Days	17 Days				
Oct. 8	Nov. 4	Dec. 6	Jan. 15	Mar. 4	Mar. 27	May 5	Jun. 3				

Major Work Supporting Content Additional Content

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I. Unit Overview

The unit begins with a lesson on "number puzzles" in which students are shown a number line diagram that displays numerical changes (e.g., as in grade 7 work with signed numbers) and asked to write descriptions of situations and equations that the diagram could represent. Students are then given descriptions of situations in which an unknown quantity is linearly related to a combination of known quantities and asked to determine the unknown quantities in any way they can, e.g., using diagrams or writing equations.

In the second and third sections of the unit, students write and solve equations, abstracting from contexts (MP2) to represent a problem situation, stating the meanings of symbols that represent unknowns (MP6), identifying assumptions such as constant rate (MP4), selecting methods and representations to use in obtaining a solution (MP5), reasoning to obtain a solution (MP1), interpreting solutions in the contexts from which they arose (MP2) and writing them with appropriate units (MP6), communicating their reasoning to others (MP3), and identifying correspondences between verbal descriptions, tables, diagrams, equations, and graphs, and between different solution approaches (MP1).

The second section focuses on linear equations in one variable. Students analyze "hanger diagrams" that depict two collections of shapes that balance each other. Assuming that identical shapes have the same weight, they decide which actions of adding or removing weights preserve that balance. Given a hanger diagram that shows one type of shape with unknown weight, they use the diagram and their understanding of balance to find the unknown weight. Abstracting actions of adding or removing weights that preserve balance (MP7), students formulate the analogous actions for equations, using these along with their understanding of equivalent expressions to develop algebraic methods for solving linear equations in one variable. They analyze groups of linear equations in one unknown, noting that they fall into three categories: no solution, exactly one solution, and infinitely many solutions. They learn that any one such equation is false, true for one value of the variable, or (using properties of operations) true for all values of the variable. Given descriptions of real-world situations, students write and solve linear equations in one variable, interpreting solutions in the contexts from which the equations arose.

The third section focuses on systems of linear equations in two variables. It begins with activities intended to remind students that a point lies on the graph of a linear equation if and only if its coordinates make the equation true. Given descriptions of two linear relationships students interpret points on their graphs, including points on both graphs.

Students categorize pairs of linear equations graphed on the same axes, noting that there are three categories: no intersection (lines distinct and parallel, no solution), exactly one intersection (lines not parallel, exactly one solution), and same line (infinitely many solutions).

II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSLS)	Estimated Time (Blocks)
Unit 4 Pre-Unit Assessment Optional	6.EE.A.3, 6.EE.A.3, 6.EE.B.7, 7.EE.B.4.a, 6.EE.B.5	1/2
Lesson 1: Number Puzzles	8.EE.C.7	1
Lesson 2: Keeping the Equation Balanced	8.EE.C	1
Lesson 3: Balanced Moves	8.EE.C, 8.EE.C.7	1
Lesson 4: More Balanced Moves	8.EE.C, 8.EE.C.7	1
Lesson 5: Solving Any Linear Equation	8.EE.C, 8.EE.C.7	1
Lesson 6: Strategic Solving	8.EE.C, 8.EE.C.7.b	1
Lesson 7: All, Some or No Solutions	8.EE.C.7.a	1
Lesson 8: How Many Solutions?	8.EE.C.7.a	1
Lesson 9: When Are They the Same?	8.EE.C.8	1
Lesson 10: On or Off the Line?	8.EE.C.8	1
Lesson 11: On Both of the Lines	8.EE.C.8	1
Lesson 12: Systems of Equations	8.EE.C.8, 8.EE.C.8.a, 8.EE.C.8.b	1
Lesson 13: Solving Systems of Equations	8.EE.C.8, 8.EE.C.8.a	1
Lesson 14: Solving More Systems	8.EE.C.8	1
Lesson 15: Writing Systems of Equations	8.EE.C.8.c	1
Lesson 16: Solving Problems with Systems of Equations	8.EE.C.8	1
Performance Task	8.EE.C.8	1/2
Unit 4 End of Unit Assessment Optional	8.EE.C, 8.EE.C.8.b, 8.EE.C.8.a, 8.EE.C.8, 8.EE.C.7, 8.EE.C.8.c	1
Total Time		18 Blocks
Grade 8 Interim Assessment 2		1

Major Work Supporting Content Additional Content

III. Pacinę	g Calendar					
Please comp	lete the pacing cal	endar based on th	e suggested pacin	g (see Pacing Gu	ide on page 2).	
			CEMI	RFR		
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Please compl	ete the pacing cal	endar based on th	e suggested pacing	g (see Pacing Guid	le on page 2).					
	JANUARY									
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday				
			1	2	3	4				
5	6	7	8	9	10	11				
12	13	14	15	16	17	18				
19	20	21	22	23	24	25				
26	27	28	29	30	31					

IV. PARCC Assessment Evidence Statements

Type II Type I Type III NJSLS **Evidence Statement** Clarification Math Calculator Practices ? i) Tasks do not have a 8.EE.7b Solve linear equations in one variable. context. b. Solve linear equations with rational number coefficients, MP. 6 including equations whose MP. 7 No solutions require expanding expressions using the distributive property and collecting like terms Analyze and solve pairs of 8.EE.8a i) Tasks do not have a simultaneous linear equations. context. a. Understand that solutions to a system of two linear MP. 2 equations in two variables MP. 5 correspond to points of No MP. 6 intersections of their graphs, MP. 7 because points of intersection satisfy both equations simultaneously. i) An equal number of 8.EE.8b-1 Analyze and solve pairs of simultaneous linear equations. tasks have: b. Solve systems of two linear a zero coefficient, e.g., • as in the system -s + equations in two variables (3/4)t = 2, t = 6, or;algebraically non-zero whole-• number coefficients,& whole-number solutions, or; non-zero whole-• MP. 1 number coefficients, & MP. 6 at least one fraction No MP. 7 among the solutions, or; non-zero integer • coefficients (with at least one coefficient negative), or; • non-zero rational coefficients (with at least one coefficient negative and at least one coefficient a noninteger).

	· · · ·			
<u>8.EE.8b-2</u>	Analyze and solve pairs of simultaneous linear equations. b. Estimate solutions [to systems of two linear equations in two variables] by graphing the equations.	 i) An equal number of tasks have: a zero coefficient, e.g., as in the system -s + (3/4)t = 2, t = 6, or; non-zero whole-number coefficients, and whole-number solutions, or; non-zero whole-number coefficients, and at least one fraction among the solutions, or; non-zero integer coefficients (with at least one coefficient negative), or; non-zero rational coefficient negative and at least one integer). 	MP. 5 MP. 6 MP. 7	No
<u>8.EE.8b-3</u>	Analyze and solve pairs of simultaneous linear equations. b. Solve simple cases [of systems of two linear equations in two variables] by inspection. For example, $3x + 2y = 5$ and 3x + 2y = 6 have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.	 i) Tasks have whole number or integer coefficients, one coefficient in either or both equations possibly zero. ii) Equal number of tasks involve: inconsistent systems, where the inconsistency is plausibly visible by inspection as in the italicized example, or; degenerate systems (infinitely many solutions), where the degeneracy is plausibly visible by inspection, as for example in 3x + 3y = 1, 6x + 6y = 2, or; systems with a unique solution and one coefficient zero, where the solution is plausibly visible by inspection, as for example in y = 1, 3x + y = 1. 	MP. 7	No

		iii) Tasks assess solving by inspection.		
<u>8.EE.8c</u>	Analyze and solve pairs of simultaneous linear equations. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.	i) Tasks may have three equations, but students are only required to analyze two equations at a time.	MP. 1 MP. 5 MP. 6 MP. 7	No
8.EE.C.Int.1	Understand that a two- dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	 i) Tasks do not have a context. ii) Figures may be drawn in the coordinate plane, but do not include the use of coordinates. iii) Tasks require students to make connections between congruence and transformations. 	MP. 2 MP. 7	No
<u>8.C.1.2</u>	Base reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: Knowledge and skills articulated in 8.EE.8a	-	MP. 2 MP. 3 MP. 5 MP. 6 MP. 7	Yes
<u>8.C.2</u>	Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Content Scope: Knowledge and skills articulated in 8.EE.7a, 8.EE.7b, 8.EE.8b	i) Tasks may have three equations, but students are only required to analyze two equations at a time.	MP.3 MP.6	Yes

8^{th} Grade Unit 4: Linear Equations and Linear Systems

<u>8.C.4.1</u>	Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 8.EE.8c	-	MP. 1 MP. 2 MP. 3 MP. 6 MP. 7	Yes

V. Differentiated Instruction

Supporting English Language Learners

The purpose of this document is to nudge the field forward by offering support to the next generation of mathematics learners and by challenging persistent assumptions about how to support and develop students' disciplinary language. The goal is to provide guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense making. UL/SCALE provides a framework for organizing strategies and special considerations to support students in learning mathematics practices, content, and language. The framework is intended to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while the framework can and should be used to support all students learning mathematics, it is particularly well-suited to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English.

For more information, click the link below:

Supporting ELL Learners

Supporting Students with Disabilities

The philosophical stance that guided the creation of these materials is the belief that with proper structures, accommodations, and supports, all children can learn mathematics. Lessons are designed to maximize access for all students and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

For more information, click the link below:

Supporting Students with Disabilities

<u>Constant Term</u>	In an expression like 5x+2 the number 2 is called the constant term because it doesn't change when x changes.
<u>Systems of</u> Equations	A system of equations is a set of equations where you want to find a solution that makes all the equations true at the same time. In these materials it is a pair of two linear equations in two variables.

VII. Assessment Framework

Unit 4 Assessment Framework								
Assessment	NJSLS	Estimated	Format	Graded				
		Time		?				
Pre-Unit Diagnostic	6.EE.A.3, 6.EE.A.3,	1/2 Block	Individual	Yes				
Assessment	6.EE.B.7, 7.EE.B.4.a,			(No Weight)				
(Beginning of Unit – Optional)	6.EE.B.5							
Illustrative Mathematics								
End-of-Unit Assessment	8.EE.C, 8.EE.C.8.b,	1 Block	Individual	Yes				
(End of Unit – Optional)	8.EE.C.8.a,							
Illustrative Mathematics	8.EE.C.8, 8.EE.C.7,							
	8.EE.C.8.c							

Unit 4 Performance Assessment Framework								
Assessment	NJSLS	Estimated	Format	Graded 2				
Unit 4 Performance Task 1 (Mid-January) Fixing the Furnace	8.EE.C.8	¹ / ₂ Block	Individual	Yes; Rubric				
Unit 4 Performance Task Option 1 (Optional) Kimi and Jordan	8.EE.C.8	Teacher Discretion	Teacher Discretion	Yes, if administered				

8 th Grade: Unit 4 Performance Task					
Name	Block	Date			
Fixing the Furnace (8.EE.C.8)					
Ivan's furnace has quit working during the decides to call some mechanics and furn	e coldest part of the year, ace specialists to see wh	and he is eager to get it fixed. He at it might cost him to have the			

furnace fixed. Since he is unsure of the parts he needs, he decides to compare the costs based only on service fees and labor costs. Shown below are the price estimates for labor that were given to him by three different companies. Each company has given the same time estimate for fixing the furnace.

- Company A charges \$35 per hour to its customers.
- Company B charges a \$20 service fee for coming out to the house and then \$25 per hour for each additional hour.
- Company C charges a \$45 service fee for coming out to the house and then \$20 per hour for each additional hour.

For which time intervals should Ivan choose Company A, Company B, Company C? Support your decision with sound reasoning and representations. Consider including equations, tables, and graphs.

8 th Grade Unit 4: Linear Equations and Linear System	s	
8 th Grade Fixing the Furnace	Name:	Date:
<i>NJSLS</i> : 8.EE.C.8	Type:	Teacher:

SOLUTION

- a. Student models each company's cost with a graph and/or equations.b. Student accurately determines for how many hours each company has the same total charge as the others.
- c. Student accurately justifies for which number of hours each company is the least expensive.

Level 5:	Level 4:	Level 3:	Level 2:	Level 1:	
Distinguished	Strong	Moderate	Partial	No	
Command	Command	Command	Command	Command	
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.	

Answer

Let x be the number of hours it takes to fix the furnace, and y the cost in dollars of fixing the furnace. Company A's cost can be modeled with the equation y=35x, company B's with the equation y=25x+20, and company C's with the equation y=20x+45.

Graphing the equations helps students visualize the solution.



To find the solution algebraically, consider each pair of equations as a system. Let x be the number of hours it takes to repair the furnace and y be the cost of the repair (without parts).

To find the number of hours for which company A and company B cost the same, consider y=35x and y=25x+20. Substituting for y, we get 35x=25x+20. The solution to this equation gives the number of hours for which company A and company B cost the same. Solving this equation, we find that the cost of company A and company A and company B is the same for 2 hours of labor. The cost is \$70.

To find the number of hours for which company A and company C cost the same, consider y=35x and y=20x+45. Substituting for y, we get 35x=20x+45. Solving this equation, we find that the cost of company A and company C is the same for 3 hours labor, for a total cost of \$105.

To find the number of hours for which company B and company C cost the same, consider y=25x+20 and y=20x+45. Substituting for y, we get 25x+20=20x+45. Solving this equation, we find that the cost of company B and company C is the same for 5 hours labor, for a total cost of \$145.

Some additional substitution of values shows that company C is \$15 more expensive than A and B at 2 hours. Company B is \$10 less than A and C at 3 hours. And company A is \$30 more expensive at 5 hours than companies B and C.

As before, we found that company A is the least expensive up to a time of 2 hours, at which point company A and B are the same cost. From 2 hours to 5 hours, company B is the least expensive, and at 5 hours company B and C both cost \$145. For more than 5 hours, Company C will be the least expensive.

8th Grade: Unit 4 Performance Task Option 1

Name _____

Block _____

Date _____

Kimi and Jordan (8.EE.C.8)

Kimi and Jordan are each working during the summer to earn money in addition to their weekly allowance, and they are saving all their money. Kimi earns \$9 an hour at her job, and her allowance is \$8 per week. Jordan earns \$7.50 an hour, and his allowance is \$16 per week.

a) Complete the two tables shown below.

Number of hours worked in a week, h	0	1	2	3	4	5	6	7
Kimi's weekly total savings, K								
_								
Number of hours worked in a week, h	0	1	2	3	4	5	6	7
Jordan's weekly total savings, J								

- b) Write an equation that can be used to calculate the total of Kimi's allowance and job earnings at the end of one week given the number of hours she works.
- c) Write an equation that can be used to calculate the total of Jordan's allowance and job earnings at the end of one week given the number of hours worked.

d) Sketch the graphs of your two equations on one pair of axes.

e) Jordan wonders who will save more money in a week if they both work the same number of hours. Write an answer for him.

IX. 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see 21st Century Career Ready Practices .

8th Grade Unit 4: Linear Equations and Linear Systems

References

"Illustrative Mathematics" Open Up Resources. 2018

<https://auth.openupresources.org/register/complete>