8th Grade Mathematics

Linear Relationships

Unit 3 Pacing Calendar - Illustrative Mathematics

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Revised: 10/22/2019

From the New Jersey Student Learning Standards:

In **Grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Yearlong Pacing Guide
Grade 8

Grade	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
5	Unit 1 5.NBT	Unit 2 5.NBT		Unit 3 5.NF	Unit 4 5.NF		Unit 5 5.MD		Unit 6 5.OA & 5.G	
6	Unit 1 6.G	Unit 2 6.RP	Unit 3 6.RP	Unit 4 6.NS		Unit 5 6.NS	Unit 6 6.EE	Unit 7 6.NS	Unit 8 6.SP	
7	Unit 1 7.G	Unit 2 7.RP	Unit 3 7.G	Unit 4 7.RP	Unit 5 7.NS	Unit 6 7.EE		Unit 7 7.G	Unit 8 7.SP	
8	**Unit 1 8.G**	**Unit 2 8.G**	**Unit 3 8.EE**	**Unit 4 8.EE**		**Unit 5 8.F**	**Unit 6 8.SP**	**Unit 7 8.EE**	**Unit 8 8.G**	

Unit 1	**Geometry:** Rigid Transformation & Congruence	Unit 2	**Geometry:** Dilations, Similarity, and Introducing Slope	Unit 3	**Expressions & Equations:** Linear Relationships	Unit 4	**Expressions & Equations:** Linear Equations & Linear Systems
Unit 5	**Functions:** Functions and Volume	Unit 6	**Statistics & Probability:** Associations in Data	Unit 7	**Expressions & Equations:** Exponents and Scientific Notation	Unit 8	**Geometry:** Pythagorean Theorem and Irrational Numbers

2019-2020 Grade 8 (iM)							
Quarter 1		**Quarter 2**		**Quarter 3**		**Quarter 4**	
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
8.G.1(M) 8.G.2(M) 8.G.5(M)	8.G.4(M) 8.G.3(M) 8.EE.6(M)	8.EE.5(M) 8.F.4(S) 8.EE.8(M)	8.EE.7(M) 8.EE.8(M)	8.F.1(M) 8.F.2(M) 8.F.3(M) 8.F.4(S) 8.F.5(S) 8.G.9(A)	8.SP.1(S) 8.SP.2(S) 8.SP.3(S) 8.SP.4(S)	8.EE.1(M) 8.EE.3(M) 8.EE.4(M)	8.NS.2(S) 8.EE.2(M) 8.G.6(M) 8.G.7(M) 8.G.8(M) 8.NS.1(S)
20 Days	15 Days	17 Days	18 Days	25 Days	13 Days	18 Days	17 Days
Oct. 8	Nov. 4	Dec. 6	Jan. 15	Mar. 4	Mar. 27	May 5	Jun. 3

Major Work Supporting Content Additional Content

Table of Contents

I. Unit Overview

The unit begins by revisiting different representations of proportional relationships (graphs, tables, and equations), and the role of the constant of proportionality in each representation and how it may be interpreted in context (MP2).

Next, students analyze the relationship between number of cups in a given stack of cups and the height of the stack—a relationship that is linear but not proportional—in order to answer the question "How many cups are needed to get to a height of 50 cm?" They are not asked to solve this problem in a specific way, giving them an opportunity to choose and use strategically (MP5) representations that appeared earlier in this unit (table, equation, graph) or in the previous unit (equation, graph). Students are introduced to "rate of change" as a way to describe the rate per 1 in a linear relationship and note that its numerical value is the same as that of the slope of the line that represents the relationship. Students analyze another linear relationship (height of water in a cylinder vs number of cubes in the cylinder) and establish a way to compute the slope of a line from any two distinct points on the line via repeated reasoning (MP8). They learn a third way to obtain an equation for a linear relationship by viewing the graph of a line in the coordinate plane as the vertical translation of a proportional relationship (MP7).

So far, the unit has involved only lines with positive slopes and y-intercepts. Students next consider the graph of a line with a negative y-intercept and equations that might represent it. They consider situations represented by linear relationships with negative rates of change, graph these (MP4), and interpret coordinates of points on the graphs in context (MP2).

The unit concludes with two lessons that involve graphing two equations in two unknowns and finding and interpreting their solutions (MP2). Doing this involves considering correspondences among different representations (MP1), in particular, what it means for a pair of values to be a solution for an equation and the correspondence between coordinates of points on a graph and solutions of an equation.

II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSLS)	Estimated Time (Blocks)
Unit 3 Pre-Unit Assessment *Optional*	7.RP.A.2.a, 7.RP.A.3, 7.RP.A.2.c, 8.EE.B, 8.G.A.1, 8.G.A.1.c, 7.EE.B.3	½
Lesson 1: Understanding Proportional Relationships	8.EE.B	1
Lesson 2: Graphs of Proportional Relationships	8.EE.B, 8.EE.B.5	1
Lesson 3: Representing Proportional Relationships	8.EE.B, 8.EE.B.5	1
Lesson 4: Comparing Proportional Relationships	8.EE.B, 8.EE.B.5	1
Lesson 5: Introduction to Linear Relationships	8.EE.B	1
Lesson 6: More Linear Relationships	8.EE.B, 8.EE.B.5	1
Lesson 7: Representations of Linear Relationships	8.EE.B, 8.EE.B.6	1
Lesson 8: Translating to y=mx+b	8.EE.B, 8.G.A.1	1
Lesson 9: Slopes Don't Have to be Positive	8.EE.B	1
Lesson 10: Calculating Slope	8.EE.B, 8.EE.B.6	1
Lesson 11: Equations of All Kinds of Lines	8.EE.B, 8.EE.B.6	1
Lesson 12: Solutions to Linear Equations	8.EE.B, 8.EE.C	1
Lesson 13: More Solutions to Linear Equations	8.EE.C, 8.EE.C.8.a	1
Lesson 14: Using Linear Relations to Solve Problems	8.EE.B.6, 8.EE.C.8.a	1
Performance Task	8.EE.B.5	½
Unit 3 End of Unit Assessment *Optional*	8.EE.B.5, 8.F.B.4, 8.EE.B	**1**
Total Time		**16 Blocks**

Major Work Supporting Content Additional Content

III. Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2)*.

NOVEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2)*.

DECEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	**2**	**3**	**4**	**5**	**6**	**7**
8	**9**	**10**	**11**	**12**	**13**	**14**
15	**16**	**17**	**18**	**19**	**20**	**21**
22	**23**	**24**	**25**	**26**	**27**	**28**
29	**30**	**31**				

IV. PARCC Assessment Evidence Statements

Type I Type II Type III

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
8.EE.5-1	Graph proportional relationships, interpreting the unit rate as the slope of the graph.	i) Tasks may or may not have context	MP. 1 MP. 5	Yes
8.EE.5-2	Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed.	i) Tasks may or may not have context	MP. 7	Yes
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.	i) Tasks without context. ii) Given a non-vertical line in the coordinate plane, tasks might for example require students to choose two pairs of points and record the rise, run, and slope relative to each pair and verify that they are the same. iii) For the explain aspect of 8.EE.6, see 8.C.5.1. iv) Tasks may assess simple graphing of lines from a linear equation in slope-intercept form.	MP. 2 MP. 7	No
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph.	i) Tasks may or may not have a context.	MP.2 MP. 4	Yes
8.C.1.1	Base reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: Knowledge and skills articulated in 8.EE.6	i) Tasks require students to derive the equation y=mx for a line through the origin and the equation y=mx+b for a line intersecting the vertical axis at b.	MP. 2 MP. 3 MP. 7 MP. 8	Yes

8.C.5.1	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.EE.6	-	MP. 2 MP. 3 MP. 5	Yes
8.C.6	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 7.RP.A, 7.NS.A, 7.EE.A	i) Some of the tasks may use scaffolding1	MP. 3 MP. 5	Yes
8.D.1	Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 8, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.	i) Some of the tasks may use scaffolding1.	MP. 1 MP. 2 MP. 4 MP. 5 MP. 7	Yes
8.D.3	Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.	i) Some of the tasks may use scaffolding1.	MP. 1 MP. 2 MP. 4 MP. 5 MP. 7	Yes
8.D.4	Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity. Content Scope: Knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements	i) Some of the tasks may use scaffolding1.	MP. 1 MP. 2 MP. 4 MP. 5 MP. 7	Yes

V. Differentiated Instruction

Supporting English Language Learners

The purpose of this document is to nudge the field forward by offering support to the next generation of mathematics learners and by challenging persistent assumptions about how to support and develop students' disciplinary language. The goal is to provide guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense making. UL/SCALE provides a framework for organizing strategies and special considerations to support students in learning mathematics practices, content, and language. The framework is intended to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while the framework can and should be used to support all students learning mathematics, it is particularly well-suited to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English.

For more information, click the link below:

Supporting ELL Learners

Supporting Students with Disabilities

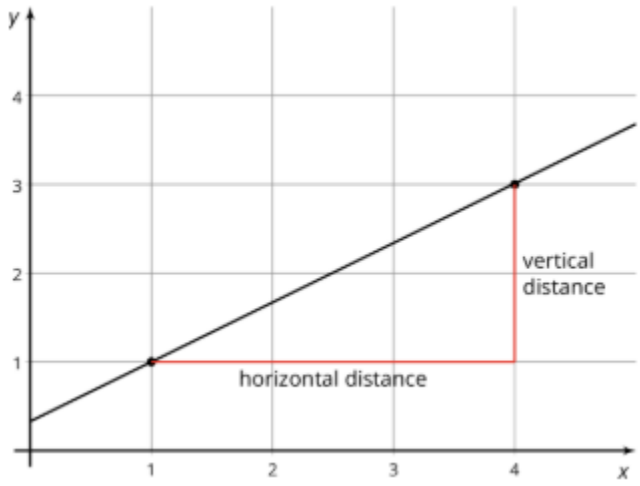
The philosophical stance that guided the creation of these materials is the belief that with proper structures, accommodations, and supports, all children can learn mathematics. Lessons are designed to maximize access for all students and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

For more information, click the link below:

Supporting Students with Disabilities

VI. Vocabulary

Linear Relationship	A linear relationship is a relationship between two quantities where one quantity has a constant rate of change with respect to the other. The relationship is called linear because its graph is a line. A linear relationship can be represented by an equation of the form y=mx+b, where m and b are constants.
Rate of Change	In a linear relationship between two quantities x and y, with equation y=mx+b, the constant m is the rate of change. It tells you how much y changes when x changes by 1. It is also the slope of the graph of the relationship.
Slope	The slope of a line is the quotient of the vertical distance and the horizontal distance between any two points on the line.



Solution to an equation with two variables	A solution to an equation with two variables is any pair (x,y) that can be used in place of the variables to make the equation true.
Vertical Intercept	The vertical intercept of a graph is the point where the graph crosses the vertical axis. If the axis is labeled with the variable y, the vertical intercept is also called the y-intercept. Also, the term is sometimes used to mean just the y-coordinate of the point where the graph crosses the vertical axis. The vertical intercept of the graph of y=3x−5 is (0,-5), or just -5.

VII. Assessment Framework

Unit 3 Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Pre-Unit Diagnostic Assessment (Beginning of Unit – Optional) *Illustrative Mathematics*	7.RP.A.2.a, 7.RP.A.3, 7.RP.A.2.c, 8.EE.B, 8.G.A.1, 8.G.A.1.c, 7.EE.B.3	½ Block	Individual	Yes (No Weight)
End-of-Unit Assessment (End of Unit – Optional) *Illustrative Mathematics*	8.EE.B.5, 8.F.B.4, 8.EE.B	1 Block	Individual	Yes

Unit 3 Performance Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 3 Performance Task 1 (Early December) *Who Has the Best Job*	8.EE.B.5	½ Block	Individual	Yes; Rubric
Unit 3 Performance Task Option 1 (Optional) *Peaches and Plums*	8.EE.B.5	Teacher Discretion	Teacher Discretion	Yes, if administered

Name _____ Block _____ Date _____

Who Has the Best Job? (8.EE.B.5)

Kell works at an after-school program at an elementary school. The table below shows how much money he earned every day last week.

	Monday	Wednesday	Friday
Time worked	1.5 hours	2.5 hours	4 hours
Money earned	$12.60	$21.00	$33.60

a. Mariko has a job mowing lawns that pays $7 per hour. Who would make more money for working 10 hours? Explain or show work.

b. Draw a graph that represents y, the amount of money Kell would make for working x hours, assuming he made the same hourly rate he was making last week.

c. Using the same coordinate axes, draw a graph that represents y, the amount of money Mariko would make for working x hours.

d. How can you see who makes more per hour just by looking at the graphs? Explain.

8th Grade Who Has the Best Job? Name: _____ Date: _____

NJSLS: 8.EE.B.5 Type:_____ Teacher: _____

SOLUTION

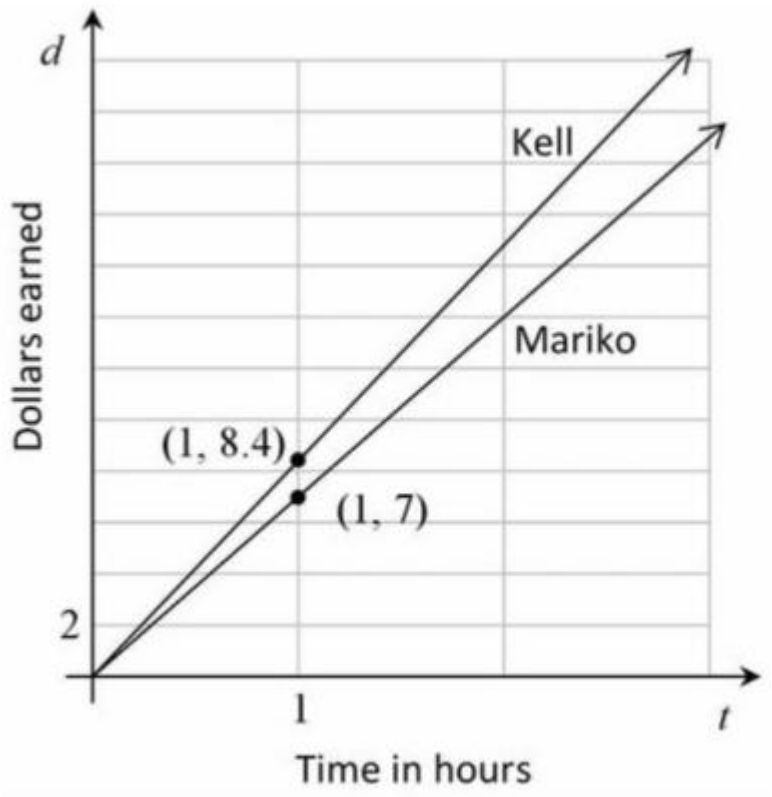
a. The student indicates Mariko will make $70.00, computes Kell's hourly rate and indicates that she will make 84.00, so she earns more money if she works 10 hours.
b. Student creates a table with all the points for both Mariko and Kell. Students graph both situations by indicating x axis as time and y axis as dollars earned.
c. The student indicates that at t = 1 Kell is making $8.40 vs. Mariko who is making $7.00. Or that Kell's line is above Mariko's line, which makes her line steeper. Or the value of the slope of Kell's line is bigger than the slope of Mariko's line.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

8th Grade Who Has the Best Job? – Scoring Guide

<table>
<tr><th></th><th>Solution</th></tr>
<tr><td>a.</td><td>
Mariko would make $7 \times 10 = 70$ dollars for working 10 hours.

Kell's hourly rate can be found by dividing the money earned by the hours worked each day.
<table>
<tr><td></td><td>Monday</td><td>Wednesday</td><td>Friday</td></tr>
<tr><td>Time worked</td><td>1.5 hours</td><td>2.5 hours</td><td>4 hours</td></tr>
<tr><td>Money earned</td><td>$12.60</td><td>$21.00</td><td>$33.60</td></tr>
<tr><td>Pay rate</td><td>$8.40 per hour</td><td>$8.40 per hour</td><td>$8.40 per hour</td></tr>
</table>
If Kell works for 10 hours at this same rate, he will earn $8.4 \times 10 = 84$ dollars. So Kell will earn more money for working 10 hours.

Alternatively, we could reason proportionally without computing the unit rate. Since Mariko earned $21.00 for 2.5 hours, she will earn four times as much for working four times as long ($10 = 4 \times 2.5$), for a total of $4 \times \$21 = \$84$.
</td></tr>
<tr><td>b.</td><td>


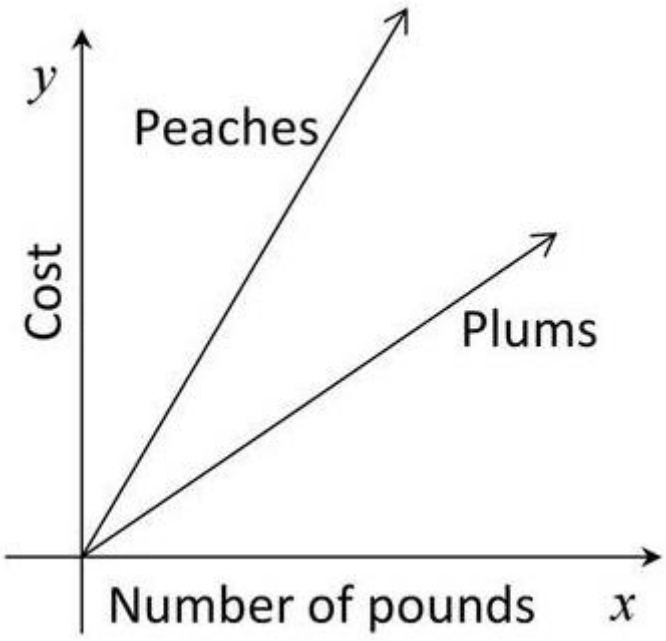
</td></tr>
<tr><td>c.</td><td>See the figure above.</td></tr>
</table>

d.	You can see that Kell will make more per hour if you look at the points on the graph where x = 1. Since this will tell you how much money each person will make for working 1 hour, you can see that Kell's line is above Mariko's line. This makes her line steeper than Mariko's line. You can also compare the slopes of the two graphs which are equal to the hourly rate.

Name _____ Block _____ Date _____

Peaches and Plums (8.EE.B.5)

The graphs below show the cost y of buying x pounds of fruit. One graph shows the cost of buying x pounds of peaches, and the other shows the cost of buying x pounds of plums.



a) Which kind of fruit costs more per pound? Explain.

b) Bananas cost less per pound than peaches or plums. Draw a line alongside the other graphs that might represent the cost y of buying x pounds of bananas.

IX. 21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Illustrative Mathematics" *Open Up Resources.* 2018

<https://auth.openupresources.org/register/complete>