8th Grade Mathematics

Dilations, Similarity, and Introducing Slope

Unit 2 Pacing Calendar - Illustrative Mathematics

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Revised: 10/22/2019

From the New Jersey Student Learning Standards:

In **Grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Yearlong Pacing Guide
Grade 8

Grade	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
5	Unit 1 5.NBT	Unit 2 5.NBT		Unit 3 5.NF	Unit 4 5.NF		Unit 5 5.MD		Unit 6 5.OA & 5.G	
6	Unit 1 6.G	Unit 2 6.RP	Unit 3 6.RP	Unit 4 6.NS		Unit 5 6.NS	Unit 6 6.EE	Unit 7 6.NS	Unit 8 6.SP	
7	Unit 1 7.G	Unit 2 7.RP	Unit 3 7.G	Unit 4 7.RP	Unit 5 7.NS	Unit 6 7.EE		Unit 7 7.G	Unit 8 7.SP	
8	**Unit 1 8.G**	**Unit 2 8.G**	**Unit 3 8.EE**	**Unit 4 8.EE**		**Unit 5 8.F**	**Unit 6 8.SP**	**Unit 7 8.EE**	**Unit 8 8.G**	

Unit 1	**Geometry:** Rigid Transformation & Congruence	Unit 2	**Geometry:** Dilations, Similarity, and Introducing Slope	Unit 3	**Expressions & Equations:** Linear Relationships	Unit 4	**Expressions & Equations:** Linear Equations & Linear Systems
Unit 5	**Functions:** Functions and Volume	Unit 6	**Statistics & Probability:** Associations in Data	Unit 7	**Expressions & Equations:** Exponents and Scientific Notation	Unit 8	**Geometry:** Pythagorean Theorem and Irrational Numbers

2019-2020 Grade 8 (iM)							
Quarter 1		**Quarter 2**		**Quarter 3**		**Quarter 4**	
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
8.G.1(M) 8.G.2(M) 8.G.5(M)	8.G.4(M) 8.G.3(M) 8.EE.6(M)	8.EE.5(M) 8.F.4(S) 8.EE.8(M)	8.EE.7(M) 8.EE.8(M)	8.F.1(M) 8.F.2(M) 8.F.3(M) 8.F.4(S) 8.F.5(S) 8.G.9(A)	8.SP.1(S) 8.SP.2(S) 8.SP.3(S) 8.SP.4(S)	8.EE.1(M) 8.EE.3(M) 8.EE.4(M)	8.NS.2(S) 8.EE.2(M) 8.G.6(M) 8.G.7(M) 8.G.8(M) 8.NS.1(S)
20 Days	15 Days	17 Days	18 Days	25 Days	13 Days	18 Days	17 Days
Oct. 8	Nov. 4	Dec. 6	Jan. 15	Mar. 4	Mar. 27	May 5	Jun. 3

Major Work Supporting Content Additional Content

Table of Contents

I. Unit Overview

In grade 8, students study pairs of scaled copies that have different rotation or mirror orientations, examining how one member of the pair can be transformed into the other, and describing these transformations.

Through activities students use and extend their knowledge of geometry and geometric measurement. Students begin the first lesson of the unit by looking at cut-out figures, first comparing them visually to determine if they are scaled copies of each other, then representing the figures in a diagram, and finally representing them on a circular grid with radial lines. They encounter the term “scale factor” and the new terms “dilation” and “center of dilation.” In the next lesson, students again use a circular grid with radial lines to understand that under a dilation the image of a circle is a circle and the image of a line is a line parallel to the original. During the rest of the unit, students draw images of figures under dilations on and off square grids and the coordinate plane. In describing correspondences between a figure and its dilation, they use the terms “corresponding points,” “corresponding sides,” and “image.” Students learn that angle measures are preserved under a dilation, but lengths in the image are multiplied by the scale factor. They learn the definition of “similar” and use properties of similar figures to justify claims of similarity or non-similarity and to reason about similar figures (MP3). Using these properties, students conclude that if two triangles have two angles in common, then the triangles must be similar. Students also conclude that the quotient of a pair of side lengths in a triangle is equal to the quotient of the corresponding side lengths in a similar triangle. This conclusion is used in the lesson that follows: students learn the terms “slope” and “slope triangle,” and use the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope (MP7). In the following lesson, students use their knowledge of slope to find an equation for a line. They will build on this initial work with slope in a subsequent grade 8 unit on linear relationships.

In this unit, several lesson plans suggest that each student have access to a geometry toolkit. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSLS)	Estimated Time (Blocks)
Unit 1 Pre-Unit Assessment *Optional*	6.NS.C.8, 7.RP.A.2, 7.RP.A.2.d, 6.NS.A.1, 7.G.A.1	½
Lesson 1: Projecting and Scaling	8.G.A	1
Lesson 2: Circular Grid	8.G.A	1
Lesson 3: Dilations with no Grid	8.G.A	1
Lesson 4: Dilations on a Square Grid	8.G.A, 8.G.A.3	1
Lesson 5: More Dilations	8.G.A, 8.G.A.3	1
Lesson 6: Similarity	8.G.A.2; 8.G.A.4	1
Lesson 7: Similar Polygons	8.G.A.2, 8.G.A.4	1
Lesson 8: Similar Triangles	8.G.A, 8.G.A.5	1
Lesson 9: Side Length Quotients in Similar Triangles	8.G.A, 8.G.A.4	1
Lesson 10: Meet Slope	8.EE.B.6	1
Lesson 11: Writing Equations for Lines	8.EE.B.6, 8.G.A	1
Lesson 12: Using Equations for Lines	8.EE.B.6	1
Lesson 13: The Shadow Knows	8.G.A.5	1
Performance Task	8.G.A.4	½
Unit 1 End of Unit Assessment *Optional*	8.G.A, 8.G.A.5, 8.EE.B.6, 8.G.A.4	**1**
Total Time		**15 Blocks**
Grade 8 Interim Assessment 1	8.G.A.1, 8.G.A.2, 8.G.A.3, 8.G.A.4, 8.G.A.5	**1**

Major Work Supporting Content Additional Content

III. Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2)*.

OCTOBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2)*.

NOVEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	**2**
3	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**
17	**18**	**19**	**20**	**21**	**22**	**23**
24	**25**	**26**	**27**	**28**	**29**	**30**

IV. PARCC Assessment Evidence Statements

Type I Type II Type III

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
8.G.1.a	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.	i) Tasks may or may not have context	MP. 3 MP. 5 MP. 8	No
8.G.1.b	Verify experimentally the properties of rotations, reflections, and translations: b. Angles are taken to angles of the same measure.	i) Tasks may or may not have context	MP. 3 MP. 5 MP. 8	No
8.G.1.c	Verify experimentally the properties of rotations, reflections, and translations: c. Parallel lines are taken to parallel lines.	i) Tasks may or may not have context	MP. 3 MP. 5 MP. 8	No
8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	i) Tasks do not have a context. ii) Figures may be drawn in the coordinate plane, but do not include the use of coordinates. iii) Tasks require students to make connections between congruence and transformations.	MP. 2 MP. 7	No
8.G.3	Describe the effect of dilations, translations, rotations, and reflections on two dimensional figures using coordinates.	i) Tasks have “thin context” to no context. ii) Tasks require the use of coordinates in the coordinate plane. iii) For items involving dilations, tasks must state center of dilation. iv) Centers of dilation can be the origin, the center of the original shape or the vertices of the original shape.	MP. 2 MP. 3 MP. 5	No

8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	i) Tasks do not have a context. ii) Figures may be drawn in the coordinate plane, but do not include the use of coordinates. iii) Tasks require students to make connections between similarity and transformations.	MP. 2 MP. 7	No
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.	i) Tasks do not have a context. ii) Given a non-vertical line in the coordinate plane, tasks might for example require students to choose two pairs of points and record the rise, run, and slope relative to each pair and verify that they are the same. iii) For the explain aspect of 8.EE.6, see 8.C.5.1. iv) Tasks may assess simple graphing of lines from a linear equation in slope-intercept form.	MP. 2 MP. 7	No
8.C.1.1	Base reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: Knowledge and skills articulated in 8.EE.6	i) Tasks require students to derive the equation y=mx for a line through the origin and the equation y=mx+b for a line intersecting the vertical axis at b.	MP. 2 MP. 3 MP. 7 MP. 8	Yes
8.C.3.2	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4	-	MP. 3 MP. 5 MP. 6	Yes

8.C.3.3	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.5	-	MP. 3 MP. 5 MP. 6	Yes
8.C.5.1	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.EE.6	-	MP. 2 MP. 3 MP. 5	Yes
8.C.5.2	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4	-	MP. 2 MP. 3 MP. 5	Yes

V. Differentiated Instruction

Supporting English Language Learners

The purpose of this document is to nudge the field forward by offering support to the next generation of mathematics learners and by challenging persistent assumptions about how to support and develop students' disciplinary language. The goal is to provide guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense making. UL/SCALE provides a framework for organizing strategies and special considerations to support students in learning mathematics practices, content, and language. The framework is intended to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while the framework can and should be used to support all students learning mathematics, it is particularly well-suited to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English.

For more information, click the link below:

Supporting ELL Learners

Supporting Students with Disabilities

The philosophical stance that guided the creation of these materials is the belief that with proper structures, accommodations, and supports, all children can learn mathematics. Lessons are designed to maximize access for all students and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

For more information, click the link below:

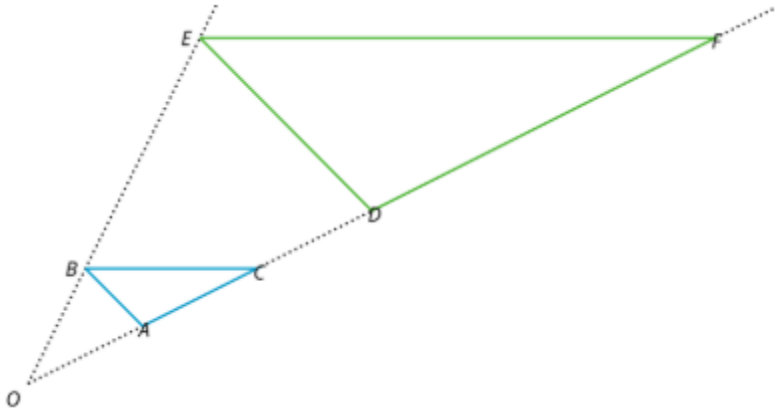
Supporting Students with Disabilities

VI. Vocabulary

Dilation

A dilation with center *O* and positive scale factor *r* takes a point *P* along the line *OP* to another point whose distance is *r* times further away from *O* than *P* is. If $r<1$ then the new point is really closer to *O*, not further away.

The triangle *DEF* is a dilation of the triangle *ABC* with center *O* and with scale factor 3. So *D* is 3 times further away from *O* than *A* is, *E* is 3 times further away from *O* than *B* is, and *F* is 3 times further away from *O* than *C* is.

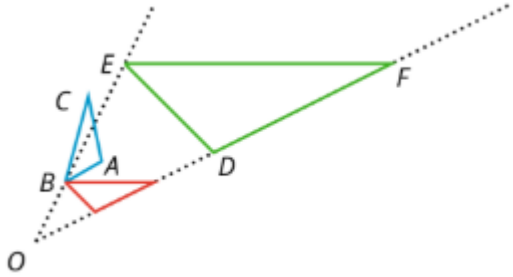


Slope:

The slope of a line is the quotient of the vertical distance and the horizontal distance between any two points on the line.

Similarity: One figure is similar to another if there is a sequence of rigid transformations and dilations that moves the first figure so that it fits exactly over the second.

Triangle *ABC* is similar to triangle *DEF* because a rotation about *B* followed by a dilation with center *O* takes the first triangle to the second.



VII. Assessment Framework

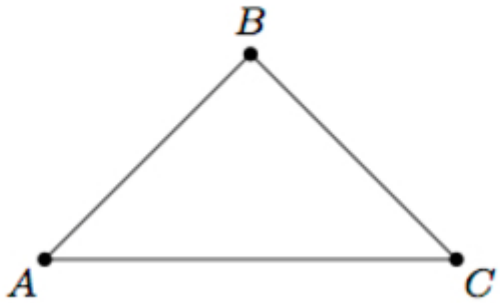
Unit 2 Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Pre-Unit Diagnostic Assessment (Beginning of Unit – Optional) *Illustrative Mathematics*	6.NS.C.8, 7.RP.A.2, 7.RP.A.2.d, 6.NS.A.1, 7.G.A.1	½ Block	Individual	Yes (No Weight)
End-of-Unit Assessment (End of Unit – Optional) *Illustrative Mathematics*	8.G.A, 8.G.A.5, 8.EE.B.6, 8.G.A.4	1 Block	Individual	Yes
Grade 8 Interim Assessment 1 (Early November) *iReady Standards Mastery*	8.G.A.1, 8.G.A.2, 8.G.A.3, 8.G.A.4, 8.G.A.5	1 Block	Individual	Yes

Unit 2 Performance Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 2 Performance Task 1 (Early November) *Creating Similar Triangles*	8.G.A.4	½ Block	Individual	Yes; Rubric
Unit 2 Performance Task Option 1 (Optional) *Are They Similar?*	8.G.A.4	Teacher Discretion	Teacher Discretion	Yes, if administered

Name _____ Block _____ Date _____

Creating Similar Triangles (8.G.A.4)

In triangle ABC below, ∠B is a right angle and |AB|=|BC|:



Draw a line segment joining one of the vertices of △ABC to the opposite side so that it divides △ABC into two triangles which are both similar to △ABC. Explain, using rigid motions and dilations, why the triangles are similar.

8th Grade Creating Similar Triangles Name: _____ Date: _____

NJSLS: 8.G.A.4 Type:_____ Teacher: _____

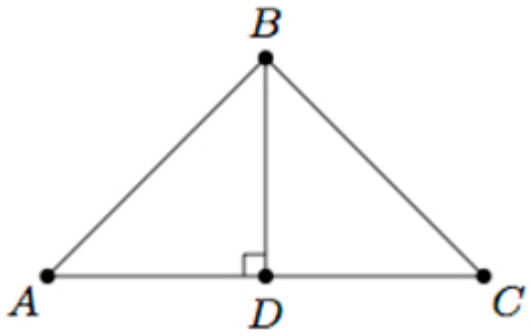
SOLUTION

a. Student draws a line segment from the vertex B that is perpendicular to the line AC.
b. Student accurately dilates and rotates the new triangle to show that the triangles are similar.
c. Student accurately justifies their findings by explaining that the rigid motions do not change the angle measures. Since the angle measurements of both triangles are 45, 45, 90, they are in fact similar.

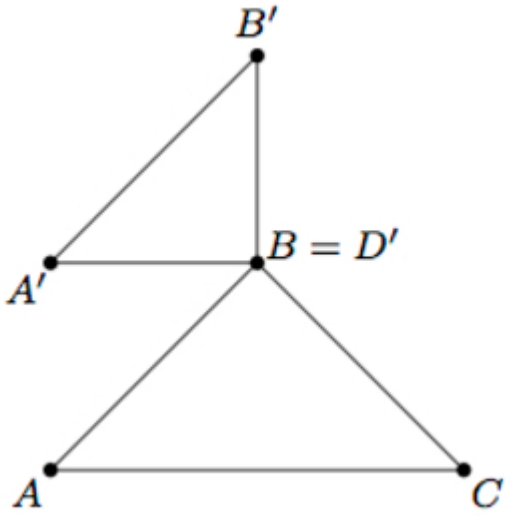
Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

8th Grade Creating Similar Triangles – Scoring Guide

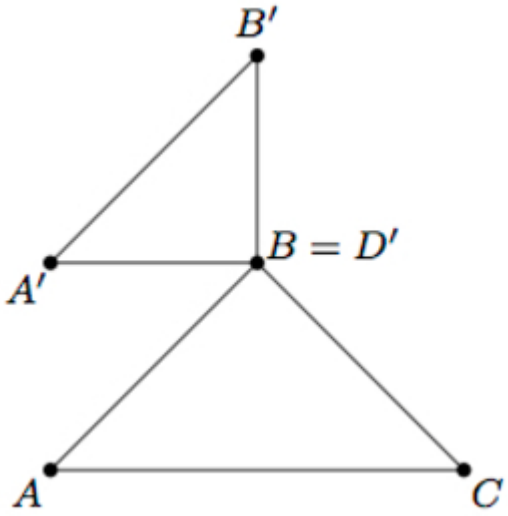
Solution
Since △ABC is a right triangle, for our two smaller triangles to be similar to it, they will also need to be right triangles. Our line must start from vertex B, and our line segment must be the one starting at B and perpendicular to line AC. In this example we have labeled the point on line AC as B so that line BD is perpendicular to line AC, shown below:



We can show that △ADB is similar to △ABC (the argument for △CDB is much the same). We can translate D to B and then rotate counterclockwise about B so that the right angle ADB matches up with the right angle CBA. We can then dilate the rotated triangle about B. We can move vertex D to match up with vertex B by translating along segment DB. The effect of this is pictured below, with the translated image of △ADB being denoted △A′D′B′:



Next we apply a rotation, about B, through angle A′BC. We denote the image of △A′D′B′ under the rotation as △A′′D′′B′′. The rotation will send line segment D′A′ to BC as shown below:

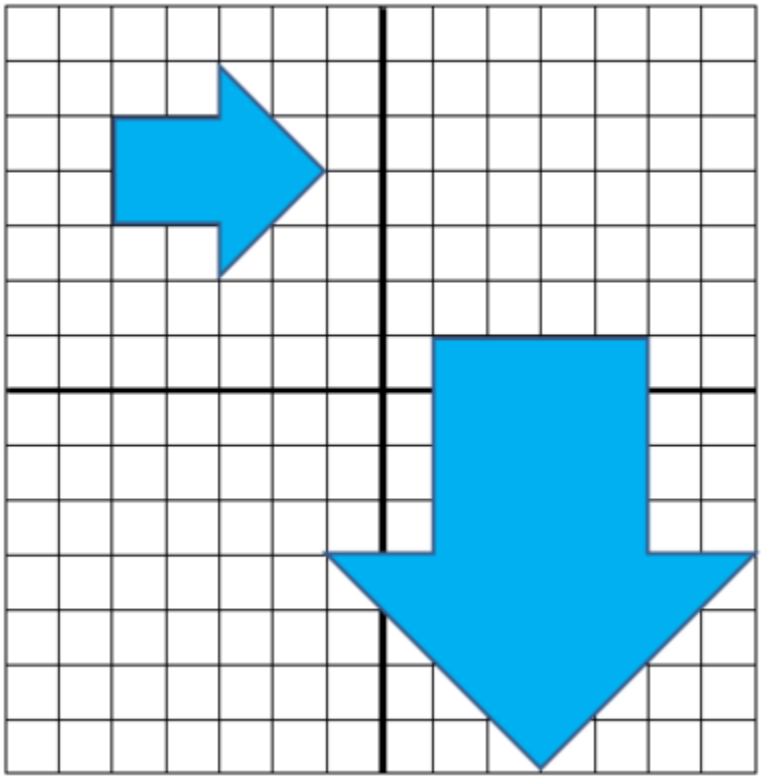


We now apply a dilation with center B which maps A'' to C and B'' to A, which finishes the argument.

Name _____ Block _____ Date _____

Are They Similar? (8.G.A.4)

Determine, using rotations, translations, reflections, and/or dilations, whether the two polygons below are similar.



The intersection of the dark lines on the coordinate plane represents the origin (0,0) in the coordinate plane.

IX. 21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Illustrative Mathematics" *Open Up Resources.* 2018

<https://auth.openupresources.org/register/complete>