# 8th Grade Mathematics

Rigid Transformations and Congruence Unit 1 Pacing Calendar - Illustrative Mathematics



# ORANGE PUBLIC SCHOOLS OFFICE OF CURRICULUM AND INSTRUCTION OFFICE OF MATHEMATICS

Revised: 08/28/2019

### From the New Jersey Student Learning Standards:

In **Grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m·A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

### Yearlong Pacing Guide Grade 8

Grade	SEP	0	СТ	N	VC	D	EC JA	N	FE	В	MA	R	A	R	Μ	AY	JUN
5	Unit 1 5.NBT		Un 5.N	it 2 IBT			Unit 3 5.NF		Un 5.1	it 4 NF		l	Jnit 5 5.MD			Unit 6 5.OA & 5.	G
6	Unit 1 6.G		Un 6.1	it 2 RP	Un 6.1	it 3 RP	Unit 4 6.NS			Unit 5 6.NS		Unit 6.E	E E	Unit 6.N	:7 S	Unit 6.S	: 8 P
7	Unit 1 7.G	Uni 7.F	it 2 RP	Un 7.	it 3 G		Unit 4 7.RP	Un 7.	it 5 NS	Ur 7	nit 6 '.EE			Unit 7 7.G		Unit 8 7.SP	
8	Unit 1 8.G	L	Un 8.	it 2 .G	Un 8.	it 3 EE	Unit 4 8.EE			Unit 5 8.F		Unit 8.S	t6 P	Unit 8.E	: 7 E	Unit 8 8.G	
	Unit 1	Geom Transi & Cor	ietry: I forma ngruen	Rigid tion Ice	Unit 2		Geometry: Dilations, Similarity, ar Introducing Slope	nd	Unit 3	t Ex Ec Ri	xpress quatic elation	sions ons: L nship	& inear s	Unit 4		Expression Equations Equations Linear Sys	ns & s: Linear ; & stems
	Unit 5	Funct Volum	ions: ions a ne	ind	Unit 6		Probability: Associations Data	in	Unit 7	E E E S N	<b>xpress</b> quatic xpone cientif lotatio	sions ons: ents a fic on	<b>a</b> nd	Unit 8		Pythagore Theorem Irrational Numbers	/: ean and

	2019-2020 Grade 8 (iM)							
Quarter 1		Quarter 2		Quar	rter 3	Quarter 4		
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	
8.G.1(M) 8.G.2(M) 8.G.5(M)	8.G.4(M) <mark>8.G.3(M)</mark> 8.EE.6(M)	8.EE.5(M) 8.F.4(S) 8.EE.8(M)	8.EE.7(M) 8.EE.8(M)	8.F.1(M) 8.F.2(M) 8.F.3(M) 8.F.4(S) 8.F.5(S) 8.G.9(A)	8.SP.1(S) 8.SP.2(S) 8.SP.3(S) 8.SP.4(S)	8.EE.1(M) 8.EE.3(M) 8.EE.4(M)	8.NS.2(S) 8.EE.2(M) 8.G.6(M) 8.G.7(M) 8.G.8(M) 8.NS.1(S)	
20 Days	15 Days	17 Days	18 Days	25 Days	13 Days	18 Days	17 Days	
Oct. 8	Nov. 4	Dec. 6	Jan. 15	Mar. 4	Mar. 27	May 5	Jun. 3	

Major Work Supporting Content Additional Content

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# I. Unit Overview

In grade 8, students extend their reasoning to plane figures with different rotation and mirror orientations.

Through activities designed and sequenced to allow students to make sense of problems and persevere in solving them (MP1), students use and extend their knowledge of geometry and geometric measurement. They begin the unit by looking at pairs of cartoons, each of which illustrates a translation, rotation, or reflection. Students describe in their own words how to move one cartoon figure onto another. As the unit progresses, they solidify their understanding of these transformations, increase the precision of their descriptions (MP6), and begin to use associated terminology, recognizing what determines each type of transformation, e.g., two points determine a translation. They identify and describe translations, rotations, and reflections, and sequences of these. In describing images of figures under rigid transformations on and off square grids and the coordinate plane, students use the terms "corresponding points," "corresponding sides," and "image." Students learn that angles and distances are preserved by any sequence of translations, rotations, and reflections, and that such a sequence is called a "rigid transformation." They learn the definition of "congruent": two figures are said to be congruent if there is a rigid transformation that takes one figure to the other. Students experimentally verify the properties of translations, rotations, and reflections, and use these properties to reason about plane figures, understanding informal arguments showing that the alternate interior angles cut by a transversal have the same measure and that the sum of the angles in a triangle is 180°. The latter will be used in a subsequent grade 8 unit on similarity and dilations. Throughout the unit, students discuss their mathematical ideas and respond to the ideas of others (MP3, MP6).

Many of the lessons in this unit ask students to work on geometric figures that are not set in a real-world context. This design choice respects the significant intellectual work of reasoning about area. Tasks set in real-world contexts are sometimes contrived and hinder rather than help understanding. Moreover, mathematical contexts are legitimate contexts that are worthy of study. Students do have opportunities in the unit to tackle real-world applications. In the culminating activity of the unit, students examine and create different patterns formed by plane figures. This is an opportunity for them to apply what they have learned in the unit (MP4).

## II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSLS)	Estimated Time (Blocks)
Unit 1 Pre-Unit Assessment Optional	8.G.A.1, 4.G.A.1, 5.G.A.1, 6.G.A.3, 7.G.B.5, 7.G.A.2, 6.G.A.1 <mark>, 8.G.A.2</mark>	1/2
Lesson 1: Moving in the Plane	8.G.A.1	1
Lesson 2: Naming the Moves	8.G.A.1	1
Lesson 3: Grid Moves	8.G.A.1	1
Lesson 4: Making the Moves	8.G.A.1	1
Lesson 5: Coordinate Moves	8.G.A.3	1
Lesson 6: Describing Transformations	8.G.A.1; 8.G.A.3	1
Lesson 7: No Bending or Stretching	8.G.A.1.a, 8.G.A.1.b	1
Lesson 8: Rotation Patterns	8.G.A.1.a, 8.G.A.1.b	1
Lesson 9: Moves in Parallel	8.G.A.1.a, 8.G.A.1.b, 8.G.A.1.c	1
Lesson 10: Composing Figures	8.G.A.1.a, 8.G.A.1.b	1
Unit 1 Mid-Unit Assessment Optional	8.G.A.1, 8.G.A.1.a, 8.G.A.1.b, 8.G.A.3	1
Lesson 11: What is the Same?	8.G.A.1, 8.G.A.2	1
Lesson 12: Congruent Polygons	8.G.A.2	1
Lesson 13: Congruence	8.G.A.1.a, 8.G.A.2	1
Lesson 14: Alternate Interior Angles	8.G.A.1, 8.G.A.5	1
Lesson 15: Adding the Angles in a Triangle	8.G.A.2, 8.G.A.5.	1
Lesson 16: Parallel Lines and the Angles in a Triangle	8.G.A.5	1
Lesson 17: Rotate and Tessellate	8.G.A	1
Performance Task	8.G.A.1	1/2
Unit 1 End of Unit Assessment Optional	8.G.A.1, 8.G.A.2, 8.G.A.1.a, 8.G.A.5	1
Total Time		20 Blocks

Major Work Supporting Content Additional Content

### III. Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (see Pacing Guide on page 2).

		SFP		BFR		
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

Please complete the pacing calendar based on the suggested pacing (see Pacing Guide on page 2).

		OC	TOB	ER		
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

### IV. PARCC Assessment Evidence Statements

Type I Type II Type III

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
<u>8.G.1.a</u>	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.	i) Tasks may or may not have context	MP. 3 MP. 5 MP. 8	No
<u>8.G.1.b</u>	Verify experimentally the properties of rotations, reflections, and translations: b. Angles are taken to angles of the same measure.	i) Tasks may or may not have context	MP. 3 MP. 5 MP. 8	No
<u>8.G.1.c</u>	Verify experimentally the properties of rotations, reflections, and translations: c. Parallel lines are taken to parallel lines.	i) Tasks may or may not have context	MP. 3 MP. 5 MP. 8	No
<u>8.G.2</u>	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	<ul> <li>i) Tasks do not have a context.</li> <li>ii) Figures may be drawn in the coordinate plane, but do not include the use of coordinates.</li> <li>iii) Tasks require students to make connections between congruence and transformations.</li> </ul>	MP. 2 MP. 7	No
<u>8.G.3</u>	Describe the effect of dilations, translations, rotations, and reflections on two dimensional figures using coordinates.	<ul> <li>i) Tasks have "thin context" to no context.</li> <li>ii) Tasks require the use of coordinates in the coordinate plane.</li> <li>iii) For items involving dilations, tasks must state center of dilation.</li> <li>iv) Centers of dilation can be the origin, the center of the original shape or the vertices of the original shape.</li> </ul>	MP. 2 MP. 3 MP. 5	No

<u>8.C.3.2</u>	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4	-	MP.3 MP.5 MP.6	Yes
<u>8.C.3.3</u>	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 8.G.5	-	MP.3 MP.5 MP.6	Yes
<u>8.C.5.2</u>	Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.G.2, 8.G.4	-	MP.2 MP.3 MP.5	Yes

# V. Differentiated Instruction

### Supporting English Language Learners

The purpose of this document is to nudge the field forward by offering support to the next generation of mathematics learners and by challenging persistent assumptions about how to support and develop students' disciplinary language. The goal is to provide guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense making. UL/SCALE provides a framework for organizing strategies and special considerations to support students in learning mathematics practices, content, and language. The framework is intended to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while the framework can and should be used to support all students learning mathematics, it is particularly well-suited to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English.

For more information, click the link below:

Supporting ELL Learners

#### Supporting Students with Disabilities

The philosophical stance that guided the creation of these materials is the belief that with proper structures, accommodations, and supports, all children can learn mathematics. Lessons are designed to maximize access for all students and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

For more information, click the link below:

Supporting Students with Disabilities

<u>Alternate</u> Interior angles are angles that are made by a transversal crossing two <u>Interior Angles:</u> parallel lines. They are the angles that lie between the parallel lines, not outside them.

If two interior angles lie on opposite sides of the transversal they are called alternate interior angles.

In the figure, *a* and *d* are alternate interior angles, and *b* and *c* are also alternate interior angles.



<u>Clockwise:</u> An object is rotating clockwise if it is turning in the same way that the hour or minute hand goes around a clock.

The tilted square is rotated 15° clockwise from the square sitting horizontally on its base.



<u>Congruent:</u> One figure is congruent to another if there is a rigid transformation (a sequence of translations, rotations, and reflections) that moves the first figure so that it fits exactly over the second. The second figure is called the image of the rigid transformation.

Triangle A is congruent to triangle D. A translation takes triangle A to triangle B, a rotation takes triangle B to triangle C, and a reflection takes triangle C to triangle D.

C B

<u>Corresponding:</u> If a part of the original figure matches up with a part of the copy, we call them corresponding parts. The part could be an angle, point, or side, and you can have corresponding angles, corresponding points, or corresponding sides.

If you have a distance between two points in the original figure, then the distance between the corresponding points in the copy is called the corresponding distance.

<u>Counterclockwise</u>: An object is rotating counterclockwise if it is turning in the opposite way to the way that the hour or minute hand goes around a clock.

The tilted square is rotated 15° counterclockwise from the square with a horizontal base.



<u>Image:</u> Translations, rotations, and reflections move objects in the plane. Points, segments, and other parts of the original all have corresponding parts on the "moved object." The moved object is called the image.

For example, here is triangle **ABC** and a translation to the right and up which is labeled **DEF**.

Point **F** in the image corresponds to point **C**, segment **EF** in the image corresponds to segment **BD**, and angle **DEF** corresponds to angle **ABC**.



Reflection:The reflection of a figure across a line takes every point of the figure to a<br/>point directly opposite to it on the other side of the line and the same<br/>distance from the line. In the figure, the triangle B is the reflection of the<br/>triangle A across the line ℓ.



### <u>Rigid</u> <u>Transformation:</u>

A rigid transformation is a sequence of translations, rotations, and reflections. If a rigid transformation is applied to a geometric figure, the resulting figure is called the image of the original figure under the transformation.

#### $\mathbf{8}^{\text{th}}$ Grade Unit 1: Rigid Transformations and Congruence

Rotation:

A rotation has a center, an angle, and a direction. It moves every point of a figure in a circle around the center, in the direction specified (clockwise or counterclockwise), and for a distance specified by the angle. For example, in the figure, triangle A is rotated 55° clockwise about center O to get triangle B.



### <u>Sequence of</u> A sequence <u>Transformations</u>: reflections

A sequence of transformations is a set of translations, rotations, reflections, and dilations performed in a particular order on a geometric figure, resulting in a final figure.

The diagram shows a sequence of transformations consisting of a translation (from A to B) followed by a rotation (from B to C) followed by a reflection (from C to D). The last triangle is the final figure resulting from the sequence.



<u>Straight Angle:</u> If the two rays that make an angle form a straight line, we call the angle a straight angle.

- <u>Transformation</u>: A transformation is a translation, rotation, reflection, or dilation, or combination of these. There is also a more general concept of a transformation of the plane that is not discussed in grade 8.
- <u>Translation:</u> A translation has a distance and a direction. It moves every point in a figure the given distance in the given direction.

The figure on the left is translated to the figure on the right in the direction from A to B, using the distance from A to B.



- <u>Transversal:</u> A transversal to two parallel lines is a line that cuts across them, intersecting each one.
- <u>Vertical Angles:</u> A pair of vertical angles is a pair of angles that are across from each other at the point where two lines intersect. There are two pairs of vertical angles.



# VII. Assessment Framework

Unit 1 Assessment Framework						
Assessment	NJSLS	Estimated Time	Format	Graded ?		
Pre-Unit Diagnostic	8.G.A.1, 4.G.A.1,	1/2 Block	Individual	Yes		
Assessment	5.G.A.1, 6.G.A.3,			(No Weight)		
(Beginning of Unit – Optional)	7.G.B.5, 7.G.A.2,					
Illustrative Mathematics	6.G.A.1, 8.G.A.2					
Mid-Unit Assessment	8.G.A.1, 8.G.A.1.a,	1 Block	Individual	Yes		
(After Lesson 10 - Optional) Illustrative Mathematics	8.G.A.1.b, 8.G.A.3					
End-of-Unit Assessment	8.G.A.1, 8.G.A.1.a,	1 Block	Individual	Yes		
(End of Unit – Optional) Illustrative Mathematics	8.G.A.2, 8.G.A.5					

Unit 1 Performance Assessment Framework						
Assessment	NJSLS	Estimated	Format	Graded		
		Time		<b>'</b>		
Unit 1 Performance Task 1 (Early October) Triangle Congruence with Congruence	8.G.A.2, 8.G.A.3	1⁄2 Block	Individual	Yes; Rubric		
Unit 1 Performance Task	8.G.A.1	Teacher	Teacher	Yes, if		
Option 1		Discretion	Discretion	administered		
(Optional)						
Reflecting a Rectangle Over a						
Diagonal						



a. Show that  $\triangle ABC$  is congruent to  $\triangle PQR$  with a reflection followed by a translation.

b. If you reverse the order of your reflection and translation in part (a) does it still map  $\triangle ABC$  to  $\triangle PQR$ ?

c. Find a second way, different from your work in part (a), to map  $\triangle ABC$  to  $\triangle PQR$  using translations, rotations, and/or reflections.

8th Grade Triangle Congruence	with Coordinates
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*NJSLS*: 8.G.A.2, 8.G.A.3

Name:	Date:

Type:\_\_\_\_\_ Teacher: \_\_\_\_\_

#### SOLUTION

- a. Student proves the two triangles are congruent with a sequence of reflections and translations.
- b. Student accurately justifies their reason for yes or no based on the reverse sequence from part (a).
- c. Student finds a different sequence of transformations than the sequence used in part (a) to prove the two triangles are congruent.

Level 5:	Level 4:	Level 3:	Level 2:	Level 1:
Distinguished	Strong	Moderate	Partial	No
Command	Command	Command	Command	Command
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

#### 8th Grade Unit 1: Rigid Transformations and Congruence 8th Grade Triangle Congruence with Coordinates – Scoring Guide











b. Suppose you have a rectangle where the line through the diagonal is a line of symmetry. Using what you know about reflections, explain why the rectangle must be a square.

# IX. 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see 21st Century Career Ready Practices .

### References

"Illustrative Mathematics" Open Up Resources. 2018

<https://auth.openupresources.org/register/complete>