

7th Grade Mathematics

Measuring Circles

Unit 3 Pacing Calendar - Illustrative Mathematics



ORANGE PUBLIC SCHOOLS
OFFICE OF CURRICULUM AND INSTRUCTION
OFFICE OF MATHEMATICS

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From the New Jersey Student Learning Standards:

In **Grade 7**, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Yearlong Pacing Guide Grade 7

Grade	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
5	Unit 1 5.NBT	Unit 2 5.NBT		Unit 3 5.NF	Unit 4 5.NF		Unit 5 5.MD	Unit 6 5.OA & 5.G		
6	Unit 1 6.G	Unit 2 6.RP	Unit 3 6.RP	Unit 4 6.NS		Unit 5 6.NS	Unit 6 6.EE	Unit 7 6.NS	Unit 8 6.SP	
7	Unit 1 7.G	Unit 2 7.RP	Unit 3 7.G	Unit 4 7.RP	Unit 5 7.NS	Unit 6 7.EE	Unit 7 7.G		Unit 8 7.SP	
8	Unit 1 8.G	Unit 2 8.G	Unit 3 8.EE	Unit 4 8.EE	Unit 5 8.F		Unit 6 8.SP	Unit 7 8.EE	Unit 8 8.G	

Unit 1	Geometry: Scale Drawings	Unit 2	Ratios & Proportional Relationships: Introducing Proportional Relationships	Unit 3	Geometry: Measuring Circles	Unit 4	Ratios & Proportional Relationships: Proportional Relationships & Percentages
Unit 5	Number System: Rational Number Arithmetic	Unit 6	Expressions & Equations: Expressions, Equations &	Unit 7	Geometry: Angles, Triangles, and Prisms	Unit 8	Statistics & Probability: Probability and Sampling

2019-2020 Grade 7 (iM)							
Quarter 1		Quarter 2		Quarter 3		Quarter 4	
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
7.G.1(A)	7.RP.2a(M) 7.RP.2b(M) 7.RP.2c(M) 7.RP.2d(M)	7.G.4(A)	7.RP.1(M) 7.RP.3(M)	7.NS.1(M) 7.NS.2(M) 7.NS.3(M)	7.EE.3(M) 7.EE.4(M) 7.EE.2(M) 7.EE.1(M)	7.G.5(A) 7.G.2(A) 7.G.3(A) 7.G.6(A)	7.SP.6(S) 7.SP.5(S) 7.SP.7(S) 7.SP.8(S) 7.SP.1(S) 7.SP.2(S) 7.SP.3(S) 7.SP.4(S)
15 Days	17 Days	13 Days	19 Days	19 Days	25 Days	19 Days	22 Days
Oct. 1	Oct. 30	Nov. 22	Jan. 8	Feb. 7	Mar. 27	May 5	Jun. 9

Major Work
Supporting Content
Additional Content

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I. Unit Overview

In this unit, students extend their knowledge of circles and geometric measurement, applying their knowledge of proportional relationships to the study of circles. They extend their grade 6 work with perimeters of polygons to circumferences of circles, and recognize that the circumference of a circle is proportional to its diameter, with constant of proportionality π . They encounter informal derivations of the relationship between area, circumference, and radius.

The unit begins with activities designed to help students come to a more precise understanding of the characteristics of a circle (MP6): a “circle” is the set of points that are equally distant from a point called the “center”; the diameter of a circle is a line segment that passes through its center with endpoints on the circle; the radius is a line segment with one endpoint on the circle and one endpoint at the center. Students identify these characteristics in a variety of contexts (MP2). They use compasses to draw circles with given diameters or radii, and to copy designs that involve circles. Using their newly gained familiarity with circumference and diameter, students measure circular objects, investigating the relationship between measurements of circumference and diameter by making tables and graphs.

The second section involves area. Students encounter two informal derivations of the fact that the area of a circle is equal to π times the square of its radius. The first involves dissecting a disk into sectors and rearranging them to form a shape that approximates a parallelogram of height r and width $2\pi r$. A second argument involves considering a disk as formed of concentric rings, “cutting” the rings with a radius, and “opening” the rings to form a shape that approximates an isosceles triangle of height r and base $2\pi \cdot 2r$.

In the third and last section, students select and use formulas for the area and circumference of a circle to solve abstract and real-world problems that involve calculating lengths and areas. They express measurements in terms of π or using appropriate approximations of π to express them numerically. In grade 8, they will use and extend their knowledge of circles and radii at the beginning of a unit on dilations and similarity.

On using the term circle. Strictly speaking, a circle is one-dimensional—the boundary of a two-dimensional region rather than the region itself. Because students are not yet expected to make this distinction, these materials refer to both circular regions (i.e., disks) and boundaries of disks as “circles,” using illustrations to eliminate ambiguity.

II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSLs)	Estimated Time (Blocks)
Unit 3 Pre-Unit Assessment (IM) <i>Optional</i>	6.EE.C.9, 6.RP.A.3, 6.EE.A.2, 4.MD.A.1, 4.MD.A.3, 6.EE.B.7, 6.G.A.1	½
Lesson 1: How Well Can You Measure?	7.RP.A.2.a	1
Lesson 2: Exploring Circles	7.G.A, 7.G.A.2	1
Lesson 3: Exploring Circumference	7.G.B.4, 7.RP.A.2, 7.RP.A.2.a	1
Lesson 4: Applying Circumference	7.G.B.4	1
Lesson 5: Circumference and Wheels	7.G.B.4	1
Lesson 6: Estimating Areas	7.G.A.1, 7.G.B, 7.G.B.6	1
Lesson 7: Exploring the Area of a Circle	7.G.A, 7.G.B, 7.G.B.4, 7.RP.A.2.a	1
Lesson 8: Relating Area to Circumference	7.G.B.4	1
Lesson 9: Applying Area of Circles	7.G.B.4	1
Lesson 10: Distinguishing Circumference and Area	7.G.B.4	1
Lesson 11: Stained-Glass Windows (<i>Project Based Learning</i>)	7.EE.B.3, 7.G.A.1, 7.G.B.4	1
Unit 3 Performance Task	7.G.B.4	½
Unit 3 End of Unit Assessment (IM)	7.G.B.4, 7.RP.A.2.b., 7.RP.A.2, 7.RP.A.3	1
Total Time		13 Blocks

Major Work Supporting Content Additional Content

III. Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2*).

NOVEMBER						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 2*).

DECEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

IV. PARCC Assessment Evidence Statements

Type I

Type II

Type III

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
<u>7.RP.2a</u>	Recognize and represent proportional relationships between quantities: a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	i) Tasks may or may not have context	MP. 2 MP. 5	Yes
<u>7.RP.2b</u>	Recognize and represent proportional relationships between quantities: b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	i) Tasks may or may not have context ii) Tasks sample equally across the listed representations (graphs, equations, diagrams, and verbal descriptions). iii) Tasks use only coordinates in Quadrant 1 and use only a positive constant of proportionality.	MP. 2 MP. 5 MP. 8	No
<u>7.RP.3-1</u>	Use proportional relationships to solve multistep ratio problems	i) Tasks will include proportional relationships that only involve positive numbers.	MP. 1 MP. 2 MP. 6	Yes
<u>7.G.4-1</u>	Know the formulas for the area and circumference of a circle and use them to solve problems.	i) Tasks may or may not have context. ii) Tasks may require answers to be written in terms of π	MP. 4 MP. 5	Yes
<u>7.G.4-2</u>	Give an informal derivation of the relationship between the circumference and area of a circle.	i) Tasks require students to identify or produce a logical conclusion about the relationship between the circumference and the area of a circle.	MP. 2 MP. 5	Yes

7th Grade Unit 3: Measuring Circles

<u>7.C.6.1</u>	Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 7.RP.2	i) Tasks use only coordinates in Quadrant 1 and use only a positive constant of proportionality	MP.2 MP.3 MP.6	Yes
<u>7.C.7.1</u>	Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 7.RP.3	i) Tasks use only coordinates in Quadrant 1 and use only a positive constant of proportionality.	MP.1 MP.3 MP.6 MP.7 MP.8	Yes

V. Differentiated Instruction

Supporting English Language Learners

The purpose of this document is to nudge the field forward by offering support to the next generation of mathematics learners and by challenging persistent assumptions about how to support and develop students' disciplinary language. The goal is to provide guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense making. UL/SCALE provides a framework for organizing strategies and special considerations to support students in learning mathematics practices, content, and language. The framework is intended to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while the framework can and should be used to support all students learning mathematics, it is particularly well-suited to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English.

For more information, click the link below:

[Supporting ELL Learners](#)

Supporting Students with Disabilities

The philosophical stance that guided the creation of these materials is the belief that with proper structures, accommodations, and supports, all children can learn mathematics. Lessons are designed to maximize access for all students and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

For more information, click the link below:

[Supporting Students with Disabilities](#)

VI. Vocabulary

Area of a Circle: The area of a circle whose radius is r units is πr^2 square units.

Circle: A circle of radius r with center O is the set of all points that are a distance r units from O .

To draw a circle of radius 3 and center O , use a compass to draw all the points at a distance 3 from O .

Circumference: The circumference of a circle is the distance around the circle. If you imagine the circle as a piece of string, it is the length of the string. If the circle has radius r then the circumference is $2\pi r$.

Diameter: A line segment that has endpoints on a circle and passes through the center is called a diameter of the circle. The length of this segment is also called the diameter.

Radius: The distance from the center of a circle to any point on the circle as well as, the corresponding line segment from the center to a point on the circle.

Pi (π): The Greek letter π (pronounced "pie") stands for the number that is the constant of proportionality between the circumference of a circle and its diameter. If d is the diameter and C is the circumference, then $C=\pi d$.

VII. Assessment Framework

Unit 3 Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Pre-Unit Diagnostic Assessment (Beginning of Unit – Optional) <i>Illustrative Mathematics</i>	6.EE.C.9, 6.RP.A.3, 6.EE.A.2, 4.MD.A.1, 4.MD.A.3, 6.EE.B.7, 6.G.A.1	½ Block	Individual	Yes (No Weight)
End-of-Unit Assessment (End of Unit – Optional) <i>Illustrative Mathematics</i>	7.G.B.4, 7.RP.A.2.b., 7.RP.A.2, 7.RP.A.3	1 Block	Individual	Yes

Unit 3 Performance Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Unit 3 Performance Task 1 (Late November) <i>Eight Circles</i>	7.G.B.4	½ Block	Individual	Yes; Rubric
Unit 3 Performance Task Option 1 (Optional) <i>Floor Plan 2</i>	7.G.B.1 , 7.RP.A.3	Teacher Discretion	Teacher Discretion	Yes, if administered

7th Grade: Unit 3 Performance Task

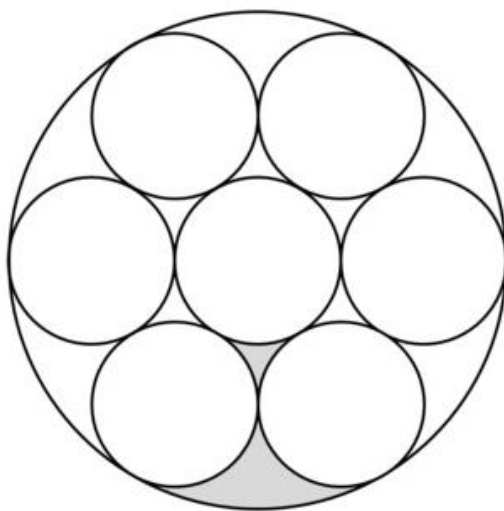
Name _____

Block _____

Date _____

Eight Circles (7.G.B.4)

The figure below is composed of eight circles, seven small circles and one large circle containing them all. Neighboring circles only share one point, and two regions between the smaller circles have been shaded. Each small circle has a radius of 5 cm.



Calculate:

- The area of the large circle.

- The area of the shaded part of the figure.

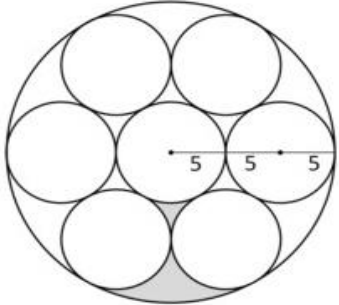
7th Grade Eight Circles - Rubric

Name: _____ Date: _____

NJSLS: 7.G.B.4

Type: _____ Teacher: _____

<p>Task Description</p>	<ul style="list-style-type: none"> • Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams. • Clearly constructs and communicates a complete response by <ul style="list-style-type: none"> ➤ using a logical approach based on a conjecture and/or stated assumptions ➤ providing an efficient and logical progression of steps ➤ using grade-level vocabulary, symbols, and labels ➤ providing a justification of a conclusion with minor computational error ➤ evaluating, interpreting and critiquing the validity and efficiency of others' responses 				
<p>Command Level Description</p>	<p>Level 5: <i>Distinguished Command</i></p> <p>Perform the task items accurately or with minor computation errors.</p>	<p>Level 4: <i>Strong Command</i></p> <p>Perform the task items with some non-conceptual errors.</p>	<p>Level 3: <i>Moderate Command</i></p> <p>Perform the task items with minor conceptual errors and some computation errors.</p>	<p>Level 2: <i>Partial Command</i></p> <p>Perform the task items with some errors on both math concept and computation.</p>	<p>Level 1:</p> <p>Perform the task items with serious errors on both math concept and computation.</p>
<p>Score range</p>	<p>6 pts</p>	<p>5 pts</p>	<p>4 pts</p>	<p>2-3 pts</p>	<p>0-1 pt</p>
<p>Task Score & PLD Assigned</p>					

#	Answer	Scoring
Part A	<p>**** Solutions may vary ***</p> <p>The radius of each of the seven small circles is 5 cm. This makes the radius of the large circle $3 \cdot 5 = 15$ cm.</p>  <p>Area of a Circle = πr^2 The area of the large circle is $\pi(15 \text{ cm})^2 = 225\pi \text{ cm}^2$.</p>	<p>2 points: 1 point for the correct strategy and 1 point for an accurate calculation</p> <p>2 TOTAL POINTS</p>
Part B	<p>The area of each small circle is $\pi(5 \text{ cm})^2 = 25\pi \text{ cm}^2$.</p> <p>There are seven small circles in all, so the area of all the small circles together is $7 \times 25\pi \text{ cm}^2 = 175\pi \text{ cm}^2$.</p> <p>If we take the area of the large circle and subtract the area of the seven small circles, we will be left with all of the area contained in the large circle that is not contained in a small circle, that is, the area around the small circles. This area is $225\pi \text{ cm}^2 - 175\pi \text{ cm}^2 = 50\pi \text{ cm}^2$.</p> <p>Notice that the exact shape of the shaded region is repeated six times in the large circle. This makes the shaded region $\frac{1}{6}$ of the area that is contained in the large circle that is not contained in a small circle. Thus the shaded region has an area of $\frac{1}{6} \times 50\pi \text{ cm}^2 = 50\pi/6 \text{ cm}^2 = 25\pi/3 \text{ cm}^2$.</p>	<p>1 point: for correctly calculating the area of each small circle.</p> <p>1 point: for correctly calculating the area of all 7 small circles</p> <p>1 point: for subtracting the total area of the small circles from the area of the large circle.</p> <p>1 point: dividing the remaining area by 6 or multiplying it by $\frac{1}{6}$.</p> <p>4 TOTAL POINTS</p>

IX. 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see [21st Century Career Ready Practices](#) .

References

“Illustrative Mathematics” *Open Up Resources*. 2018

<<https://auth.openupresources.org/register/complete>>