

6th Grade Mathematics

Unit Rates & Percentages

Unit 3 Pacing Calendar – Illustrative Mathematics



ORANGE PUBLIC SCHOOLS
OFFICE OF CURRICULUM AND INSTRUCTION
OFFICE OF MATHEMATICS

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From the New Jersey Student Learning Standards:

In **Grade 6**, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data New Jersey Student Learning Standards for Mathematics 40 distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Yearlong Pacing Guide

Grade 6

Grade	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
5	Unit 1 5.NBT	Unit 2 5.NBT		Unit 3 5.NF	Unit 4 5.NF		Unit 5 5.MD	Unit 6 5.OA & 5.G		
6	Unit 1 6.G	Unit 2 6.RP	Unit 3 6.RP	Unit 4 6.NS		Unit 5 6.NS	Unit 6 6.EE	Unit 7 6.NS	Unit 8 6.SP	
7	Unit 1 7.G	Unit 2 7.RP	Unit 3 7.G	Unit 4 7.RP	Unit 5 7.NS	Unit 6 7.EE	Unit 7 7.G		Unit 8 7.SP	
8	Unit 1 8.G	Unit 2 8.G	Unit 3 8.EE	Unit 4 8.EE	Unit 5 8.F		Unit 6 8.SP	Unit 7 8.EE	Unit 8 8.G	

Unit 1	Geometry: Area and Surface Area	Unit 2	Ratios & Proportional Relationships: Introducing Ratios	Unit 3	Ratios & Proportional Relationships: Unit Rates & Percentages	Unit 4	Number System: Dividing Fractions
Unit 5	Number System: Arithmetic in Base Ten	Unit 6	Expressions & Equations: Expressions & Equations	Unit 7	Number System: Rational Numbers	Unit 8	Statistics & Probability: Data Sets and Distributions

2019-2020 Grade 6 (iM)							
Quarter 1		Quarter 2		Quarter 3		Quarter 4	
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
6.G.1(S) 6.G.4(S)	6.RP.1(M) 6.RP.3a(M)	6.RP.2(M) 6.RP.3(M) 6.RP.3b(M) 6.RP.3c(M) 6.RP.3d(M)	6.NS.1(M) 6.G.2(S)	6.NS.3(A) 6.NS.2(A)	6.EE.6(M) 6.EE.5(M) 6.EE.7(M) 6.EE.4(M) 6.EE.2(M) 6.EE.3(M) 6.EE.1(M) 6.EE.9(M)	6.NS.5(M) 6.NS.6(M) 6.NS.7(M) 6.EE.8(M) 6.NS.8(M) 6.NS.4(A) 6.G.3(S)	6.SP.1(A) 6.SP.5(A) 6.SP.4(A) 6.SP.2(A) 6.SP.3(A)
22 Days	19 Days	19 Days	20 Days	18 Days	20 Days	20 Days	21 Days
Oct. 11	Nov. 15	Dec. 19	Jan. 31	Mar. 6	Apr. 9	May 19	Jun. 19

Major Work Supporting Content Additional Content

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I. Unit Overview

In the previous unit, students began to develop an understanding of ratios and rates. They started to describe situations using terms such as “ratio,” “rate,” “equivalent ratios,” “per,” “constant speed,” and “constant rate”. They understood specific instances of the idea that $a:b$ is equivalent to every other ratio of the form $sa:sb$, where s is a positive number. They learned that “at this rate” or “at the same rate” signals a situation that is characterized by equivalent ratios. Although the usefulness of ratios of the form $\frac{a}{b} : 1$ and $1 : \frac{b}{a}$ was highlighted, the term “unit rate” was not introduced.

In this unit, students find the two values $\frac{a}{b}$ and $\frac{b}{a}$ that are associated with the ratio $a:b$, and interpret them as rates per 1. For example, if a person walks 13 meters in 10 seconds at a constant rate, that means they walked at a speed of $\frac{13}{10}$ meters per 1 second and a pace of $\frac{10}{13}$ seconds per 1 meter.

Students learn that one of the two values ($\frac{a}{b}$ or $\frac{b}{a}$) may be more useful than the other in reasoning about a given situation. They find and use rates per 1 to solve problems set in contexts (MP2), attending to units and specifying units in their answers. For example, given item amounts and their costs, which is the better deal? Or given distances and times, which object is moving faster? Measurement conversions provide other opportunities to use rates.

Students observe that if two ratios $a : b$ and $c : d$ are equivalent, then $\frac{a}{b} = \frac{c}{d}$. The values $\frac{a}{b}$ and $\frac{c}{d}$ are called unit rates because they can be interpreted in the context from which they arose as rates per unit. Students note that in a table of equivalent ratios, the entries in one column are produced by multiplying a unit rate by the corresponding entries in the other column. Students learn that “percent” means “per 100” and indicates a rate. Just as a unit rate can be interpreted in context as a rate per 1, a percentage can be interpreted in the context from which it arose as a rate per 100. For example, suppose a beverage is made by mixing 1 cup of juice with 9 cups of water. The percentage of juice in 20 cups of the beverage is 2 cups and 10 percent of the beverage is juice. Interpreting the 10 as a rate: “there are 10 cups of juice per 100 cups of beverage” or, more generally, “there are 10 units of juice per 100 units of beverage.” The percentage—and the rate—indicate equivalent ratios of juice to beverage, e.g., 2 cups to 20 cups and 10 cups to 100 cups.

In this unit, tables and double number line diagrams are intended to help students connect percentages with equivalent ratios, and reinforce an understanding of percentages as rates per 100. Students should internalize the meaning of important benchmark percentages, for example, they should connect “75% of a number” with “ $\frac{3}{4}$ times a number” and “0.75 times a number.” Note that 75% (“seventy-five per hundred”) does not represent a fraction or decimal (which are numbers), but that “75% of a number” is calculated as a fraction of or a decimal times the number.

Work done in grades 4 and 5 supports learning about the concept of a percentage. In grade 5, students understand why multiplying a given number by a fraction less than 1 results in a product that is less than the original number, and why multiplying a given number by a fraction greater than 1 results in a product that is greater than the original number. This understanding of multiplication as scaling comes into play as students interpret, for example,

- 35% of 2 cups of juice as $\frac{35}{100} \cdot 2$ cups of juice.
- 250% of 2 cups of juice as $\frac{250}{100} \cdot 2$ cups of juice.

II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSL)	Estimated Time (Blocks)
Unit 3 Pre-Unit Diagnostic Assessment <i>Optional</i>	2.MD.A.1, 3.MD.A.2, 4.MD.A.1, 5.NBT.B.7, 5.NBT.B.5, 4.NF.C.6	1/2
Lesson 1: The Burj Khalifa	6.RP.A.2	1
Lesson 2: Anchoring Units of Measurement	6.RP.A.3d	1
Lesson 3: Measuring with Different-Sized Units	6.RP.A.3d	1
Lesson 4: Converting Units	6.RP.A.3d	1
Lesson 5: Comparing Speeds and Prices	6.RP.A.2; 6.RP.A.3b	1
Lesson 6: Interpreting Rates	6.RP.A.2; 6.RP.A.3b	1
Lesson 7: Equivalent Ratios Have the Same Unit Rates	6.RP.A.2; 6.RP.A.3b	1
Lesson 8: More about Constant Speed	6.RP.A.3; 6.RP.A.3b	1
Lesson 9: Solving Rate Problems	6.RP.A.3; 6.RP.A.3b; 6.RP.A.3d	1
Lesson 10: What Are Percentages	6.RP.A.3c	1
Lesson 11: Percentages and Double Number Line	6.RP.A.3c	1
Lesson 12: Percentages and Tape Diagrams	6.RP.A.3c	1
Lesson 13: Benchmark Percentages	6.RP.A.3c	1
Lesson 14: Solving Percentage Problems	6.RP.A.3c	1
Lesson 15: Finding This Percent of That	6.RP.A.3; 6.RP.A.3c	1
Lesson 16: Finding the Percentage	6.RP.A.3c	1
Lesson 17: Painting a Room	6.RP.A.; 6.G.A	1
Unit 3 End-of-Unit Assessment <i>Optional</i>	6.RP.A.2; 6.RP.A.3; 6.RP.A.3b; 6.RP.A.3c; 6.RP.A.3d	1
Performance Task		1 / 2
Total Time		19 Blocks

Major Work Supporting Content Additional Content

III. Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 1*).

NOVEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 1*).

DECEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

IV. PARCC Assessment Evidence Statements

Type I

Type II

Type III

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
<u>6.RP.2</u>	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger." ¹	i) Expectations for unit rates in this grade are limited to non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2	No
<u>6.RP.3b</u>	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?	i) Expectations for unit rates in this grade are limited to non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2 MP.5 MP.8	Yes
<u>6.RP.3c-1</u>	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity);	i) Tasks may or may not contain context. ii) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2 MP.5 MP.7 MP.8	Yes
<u>6.RP.3c-2</u>	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Solve problems involving finding the whole, given a part & the percent.	i) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2 MP.5 MP.7 MP.8	Yes

6th Grade Unit 3: Unit Rate and Percentages

<p><u>6.RP.3d</u></p>	<p>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p>	<p>i) Tasks may or may not contain context. ii) Tasks require students to multiply and/or divide dimensioned quantities. iii) Half of the tasks require students to correctly express the units of the result. iv) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.</p>	<p>MP.2 MP.5 MP.6 MP.7 MP.8</p>	<p>Yes</p>
<p><u>6.C.8.1</u></p>	<p>Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 6.RP.A</p>	<p>i) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.</p>	<p>MP.1 MP.2 MP.4 MP.5 MP.7</p>	<p>Yes</p>
<p><u>6.D.1</u></p>	<p>Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 6, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.</p>	<p>i) Tasks may have scaffolding, if necessary, in order yield a degree of difficulty appropriate to Grade 6.</p>	<p>MP.1 MP.2 MP.4 MP.5 MP.7</p>	<p>Yes</p>

V. Differentiated Instruction

Supporting English Language Learners

There are four design principles for promoting mathematical language use and development in curriculum and instruction. The design principles and related routines work to make language development an integral part of planning and delivering instruction while guiding teachers to amplify the most important language that students are expected to bring to bear on the central mathematical ideas of each unit.

The design principles are:

- Design Principle 1: Support sense-making
- Design Principle 2: Optimize output
- Design Principle 3: Cultivate conversation
- Design Principle 4: Maximize linguistic and cognitive meta-awareness

These four principles are intended as guides for curriculum development and planning and execution of instruction, including the structure and organization of interactive opportunities for students, and the observation, analysis, and reflection on student language and learning. The design principles motivate the use of mathematical language routines (MLRs).

These eight routines are:

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR3: Critique, Correct, and Clarify
- MLR4: Information Gap
- MLR5: Co-Craft Questions and Problems
- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Supporting Students with Disabilities

Lessons are designed to maximize access for all students, and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

The inclusion of additional supports for students with disabilities offers additional strategies for teachers to meet the individual needs of a diverse group of learners. Lesson and activity-level supports for students with disabilities are aligned to an area of cognitive functioning and are paired with a suggested strategy aimed to increase access and eliminate barriers. These lesson specific supports help students succeed with a specific activity without reducing the mathematical demand of the task. All of the supports can be used discreetly and are designed to be used as needed.

Suggestions for supports fall under the following categories:

- Eliminate Barriers
- Processing Time
- Peer Tutors
- Assistive Technology
- Visual Aids
- Graphic Organizers
- Brain Breaks

For a more descriptive account of these supports, reference the following:

https://im.openupresources.org/6/teachers/teacher_course_guide.html-supporting-students-with-disabilities

VI. VOCABULARY

Term	Definition
<i>Unit Rate</i>	A unit rate tells how many of one quantity there are per unit of another quantity.
<i>Pace</i>	Pace is an attribute that tells you how fast or slow an object is moving. A greater pace means the object is moving more slowly. Pace is measured in units like seconds per meter, hours per miles, and minutes per kilometer.
<i>Speed</i>	Speed is an attribute that tells you how fast or slow an object is moving. A greater speed means the object is moving faster. Speed is measured in units of distance per unit of time like meters per second, miles per hour, and kilometers per second.
<i>Percent</i>	The word percent means “per 100” or “for each 100” and is usually written symbolically like this: %.
<i>Percentage</i>	A percentage is a quantity described by a rate per 100.

VII. Assessment Framework

Unit 3 Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Pre-Unit Diagnostic Assessment (Beginning of Unit – Optional) <i>Illustrative Mathematics</i>	2.MD.A.1, 3.MD.A.2, 4.MD.A.1, 5.NBT.B.7, 5.NBT.B.5, 4.NF.C.6	½ Block	Individual	Yes (No Weight)
End-of-Unit Assessment (End of Unit – Optional) <i>Illustrative Mathematics</i>	6.RP.A.2; 6.RP.A.3; 6.RP.A.3b; 6.RP.A.3c; 6.RP.A.3d	1 Block	Individual	Yes

Unit 3 Performance Assessment Framework				
Assessment	NJSLS	Estimated Time	Format	Graded ?
Unit 3 Performance Task 1 (Late December) <i>Which detergent is a better buy?</i>	6.RP.A.3	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 3 Performance Task Option 1 (Optional) <i>Ratio of boys to girls</i>	6.RP.A.2 6.RP.A.3	Teacher Discretion	Teacher Discretion	Yes, if administered

6th Grade: Unit 3 Performance Task

Name _____ Block _____ Date _____

Which detergent is a better buy? (6.RP.A.3)

A store has two different brands of laundry detergent. Brand A can do 80 loads of laundry and costs \$12.75. Brand B does 36 loads of laundry and costs \$6.75. Which laundry detergent costs less per load? Show your work/ Explain your answer.

Unit 3 Performance Task 1 PLD Rubric

SOLUTION

- A) Student indicates detergent A is the better buy. Student shows work to explain their answer.

Possible Explanation: We can compare costs of the two detergents mentally. With detergent B we pay \$6.75 for 36 loads. With two detergent B's we will pay $2 \times \$6.75 = \13.50 for $2 \times 36 = 72$ loads. This is more money for fewer loads of wash compared to detergent A: \$0.75 cents more for 4 fewer loads of wash. Since detergent A does more loads of wash for less money it costs less per load of wash than detergent B.

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including:</p> <ul style="list-style-type: none"> • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error 	<p>Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including:</p> <ul style="list-style-type: none"> • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error 	<p>Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including:</p> <ul style="list-style-type: none"> • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps 	<p>Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include:</p> <ul style="list-style-type: none"> • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion 	<p>No parts are correct.</p> <p>The student shows no work or justification.</p>

6th Grade: Unit 3 Performance Task Option 1

Name _____ Block _____ Date _____

Ratio of boys to girls (6.RP.A.2)

Task

The ratio of the number of boys to the number of girls at school is 4:5.

A. What fraction of the students are boys?

B. If there are 120 boys, how many students are there altogether?

IX. 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see [21st Century Career Ready Practices](#) .

References

“Illustrative Mathematics” *Open Up Resources*. 2018

<<https://auth.openupresources.org/register/complete>>