6th Grade Mathematics

Introducing Ratios

Unit 2 Pacing Calendar – Illustrative Mathematics

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Revised: 10/16/2019

From the New Jersey Student Learning Standards:

In **Grade 6**, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

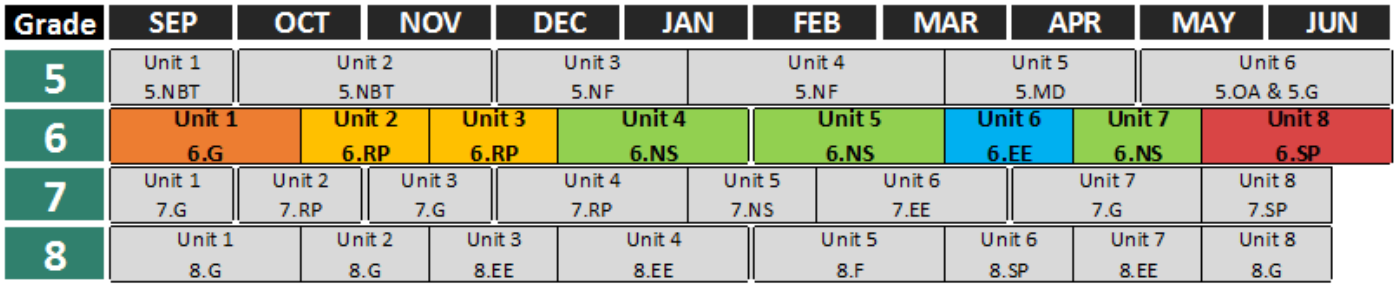
(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data New Jersey Student Learning Standards for Mathematics 40 distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Yearlong Pacing Guide
Grade 6



Unit 1 **Geometry:** Area and Surface Area

Unit 2 **Ratios & Proportional Relationships:** Introducing Ratios

Unit 3 **Ratios & Proportional Relationships:** Unit Rates & Percentages

Unit 4 **Number System:** Dividing Fractions

Unit 5 **Number System:** Arithmetic in Base Ten

Unit 6 **Expressions & Equations:** Expressions & Equations

Unit 7 **Number System:** Rational Numbers

Unit 8 **Statistics & Probability:** Data Sets and Distributions

2019-2020 Grade 6 (iM)							
Quarter 1		**Quarter 2**		**Quarter 3**		**Quarter 4**	
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
6.G.1(S) 6.G.4(S)	6.RP.1(M) 6.RP.3a(M)	6.RP.2(M) 6.RP.3(M) 6.RP.3b(M) 6.RP.3c(M) 6.RP.3d(M)	6.NS.1(M) 6.G.2(S)	6.NS.3(A) 6.NS.2(A)	6.EE.6(M) 6.EE.5(M) 6.EE.7(M) 6.EE.4(M) 6.EE.2(M) 6.EE.3(M) 6.EE.1(M) 6.EE.9(M)	6.NS.5(M) 6.NS.6(M) 6.NS.7(M) 6.EE.8(M) 6.NS.8(M) 6.NS.4(A) 6.G.3(S)	6.SP.1(A) 6.SP.5(A) 6.SP.4(A) 6.SP.2(A) 6.SP.3(A)
22 Days	19 Days	19 Days	20 Days	18 Days	20 Days	20 Days	21 Days
Oct. 11	Nov. 15	Dec. 19	Jan. 31	Mar. 6	Apr. 9	May 19	Jun. 19

Major Work Supporting Content Additional Content

Table of Contents

I. Unit Overview

In this unit, students learn that a ratio is an association between two quantities. Students analyze contexts that are often expressed in terms of ratios, such as recipes, mixtures of different paint colors, constant speed (an association of time measurements with distance measurements), and uniform pricing (an association of item amounts with prices). Equivalent ratios are first encountered in terms of multiple batches of a recipe and "equivalent" is first used to describe a perceivable sameness of two ratios. This unit introduces *discrete diagrams* and *double number line diagrams*, representations that students use to support thinking about equivalent ratios before their work with tables of equivalent ratios. After some work with double number line diagrams, students use tables to represent equivalent ratios. Use of tables to represent equivalent ratios is an important stepping-stone toward use of tables to represent linear and other functional relationships in grade 8 and beyond.

II. Pacing Guide

Activity	New Jersey State Learning Standards (NJSLS)	Estimated Time (Blocks)
Unit 2 Pre-Unit Diagnostic Assessment *Optional*	6.RP.A.1; 6.RP.A.2; 6.RP.A.3; 6.RP.A.3b	**1/2**
Lesson 1: Introducing Ratios and Ratio Language	6.RP.A.1	1
Lesson 2: Representing Ratios with Diagrams	6.RP.A.1	1
Lesson 3: Recipes	6.RP.A.1	1
Lesson 4: Color Mixtures	6.RP.A.1	1
Lesson 5: Defining Equivalent Ratios	6.RP.A.1	1
Lesson 6: Introducing Double Number Line Diagrams	6.RP.A.3	1
Lesson 7: Creating Double Number Line Diagrams	6.RP.A.3	1
Lesson 8: How much for one?	6.RP.A.3b	1
Lesson 9: Constant Speed	6.RP.A.3b	1
Lesson 10: Comparing Situations by Examining Ratios	6.RP.A.1; 6.RP.A.2; 6.RP.A.3b	1
Lesson 11: Representing Ratios with Tables	6.RP.A.3a	1
Lesson 12: Navigating a Table of Equivalent Ratios	6.RP.A.3; 6.RP.A.3a	1
Lesson 13: Tables and Double Number Lines	6.RP.A.3; 6.RP.A.3a	1
Lesson 14: Solving Equivalent Ratio Problems	6.RP.A.3	1
Lesson 15: Part-Part-Whole Ratios	6.RP.A.3	1
Lesson 16: Solving More Ratio Problems	6.RP.A.3	1
Lesson 17: A Fermi Problem	6.RP.A.; 6.RP.A.3	1
Unit 2 End-of-Unit Assessment *Optional*	6.RP.A.1; 6.RP.A.2; 6.RP.A.3; 6.RP.A.3b	**1**
Performance Task 2	6.RP.A.1	**1/2**
Total Time		**19 Blocks**
Grade 6 Interim Assessment 1	6.G.A.1, 6.G.A.4, 6.RP.A.1, 6.RP.A.3a	**1**

Major Work Supporting Content Additional Content

III. Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 1)*.

OCTOBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on page 1)*.

NOVEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

IV. PARCC Assessment Evidence Statements

Type I Type II Type III

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."	i) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2	No
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."1	i) Expectations for unit rates in this grade are limited to non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2	No
6.RP.3a	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	i) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers	MP.2 MP.4 MP.5 MP.7 MP.8	Yes

6.RP.3b	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?	i) Expectations for unit rates in this grade are limited to non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2 MP.5 MP.8	Yes
6.RP.3c-1	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity);	i) Tasks may or may not contain context. ii) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2 MP.5 MP.7 MP.8	Yes
6.RP.3c-2	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Solve problems involving finding the whole, given a part and the percent.	ii) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2 MP.5 MP.7 MP.8	Yes
6.C.8.1	Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as 1 + 4 = 5 + 7 = 12, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 6.RP.A	i) Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.	MP.2 MP.3 MP.6	Yes

V. Differentiated Instruction

Supporting English Language Learners

There are four design principles for promoting mathematical language use and development in curriculum and instruction. The design principles and related routines work to make language development an integral part of planning and delivering instruction while guiding teachers to amplify the most important language that students are expected to bring to bear on the central mathematical ideas of each unit.

The design principles are:
- Design Principle 1: Support sense-making
- Design Principle 2: Optimize output
- Design Principle 3: Cultivate conversation
- Design Principle 4: Maximize linguistic and cognitive meta-awareness

These four principles are intended as guides for curriculum development and planning and execution of instruction, including the structure and organization of interactive opportunities for students, and the observation, analysis, and reflection on student language and learning. The design principles motivate the use of mathematical language routines (MLRs).

These eight routines are:
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR3: Critique, Correct, and Clarify
- MLR4: Information Gap
- MLR5: Co-Craft Questions and Problems
- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Supporting Students with Disabilities

Lessons are designed to maximize access for all students, and include additional suggested supports to meet the varying needs of individual students. While the suggested supports are designed for students with disabilities, they are also appropriate for many children who struggle to access rigorous, grade-level content. Teachers should use their professional judgment about which supports to use and when, based on their knowledge of the individual needs of students in their classroom.

The inclusion of additional supports for students with disabilities offers additional strategies for teachers to meet the individual needs of a diverse group of learners. Lesson and activity-level supports for students with disabilities are aligned to an area of cognitive functioning and are paired with a suggested strategy aimed to increase access and eliminate barriers. These lesson specific supports help students succeed with a specific activity without reducing the mathematical demand of the task. All of the supports can be used discreetly and are designed to be used as needed.

Suggestions for supports fall under the following categories:

Eliminate Barriers
Processing Time
Peer Tutors
Assistive Technology
Visual Aids
Graphic Organizers
Brain Breaks

For a more descriptive account of these supports, reference the following:

https://im.openupresources.org/6/teachers/teacher_course_guide.html - supporting-students-with-disabilities

VI. VOCABULARY

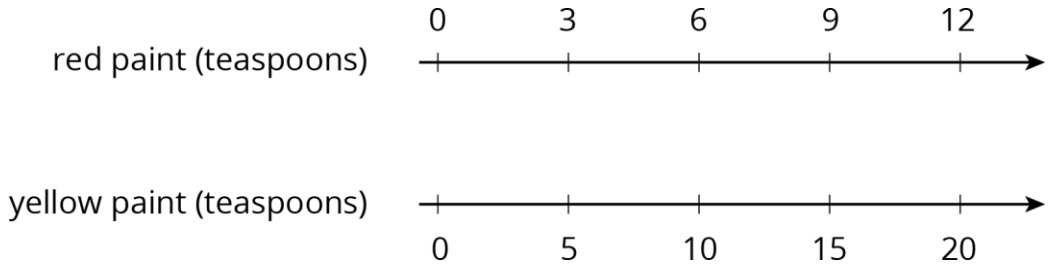
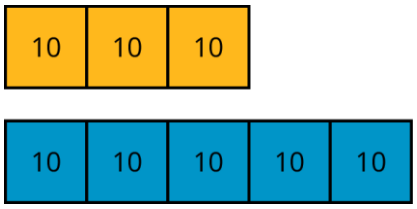
Term	Definition
Ratio	A ratio associates two or more quantities. Ratios can be described in words such as "3 to 2" and "3 for every 2" and "3 out of every 5" and "3 parts to 2 parts." We write ratios with symbols like this: 3:2.
Equivalent Ratios	Two ratios a:b and c:d are equivalent ratios if there is a number s that you can multiply both a and b by to get c and d (respectively). In other words, $a \cdot s=c$ and $b \cdot s=d$.
Double Number Line Diagram	A double number line diagram is a pair of parallel number lines with the numbers 0 aligned. Each number line is marked in equal increments and numbered. The tick marks are aligned, but the numbers on the two lines are often different. A pair of aligned numbers on the diagram represents a ratio that is equivalent to every other pair of aligned numbers on the diagram. 
Per	The word per means "for each." For example: he paid $5 for each ticket, so the cost was $5 per ticket.
Unit Price	The unit price is the cost for one item or one unit of measure.
Meters Per Second	A unit to measure speed that tells you how many meters an object travels in one second.
Same Rate	In two situations involving ratios of the same two quantities, if the ratio of the quantities in one situation is equivalent to the ratio of the quantities in the other situation then we say the two situations involve the same rate.

Table	A table is a way to organize information. Each rectangle in the table is called a cell. Each horizontal set of entries is called a row, and each vertical set of entries is called a column. The first row in a table often contains headers to explain what information is in each column. This table shows the tail-lengths of three different pets. It has four rows and two columns. The first cell in each column tells you what kind of information is in that column. pet / tail length (inches) dog / 22 cat / 12 mouse / 2
Tape Diagram	A tape diagram can be used to represent a ratio between two quantities measured in the same units. The tape diagram shows a ratio of 30 gallons of yellow paint to 50 gallons of blue paint.  A tape diagram is composed of one or more rectangles that are partitioned into equal parts. Each part represents a value. It can be any value, as long as the same value is used throughout.

VII. Assessment Framework

Unit 1 Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Pre-Unit Diagnostic Assessment (Beginning of Unit – Optional) *Illustrative Mathematics*	5.NF.B.5; 4.NF.A.1; 4.NBT.A.1; 4.OA.A.1; 4.NF.B.4b; 2.MD.B.6; 3.NF.A.2; 3.NF.A.3.b; 6.RP.A.3	½ Block	Individual	Yes (No Weight)
End-of-Unit Assessment (End of Unit – Optional) *Illustrative Mathematics*	6.RP.A.1; 6.RP.A.2 6.RP.A.3; 6.RP.A.3b	1 Block	Individual	Yes

Unit 1 Performance Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 2 Performance Task 1 (Mid- November) *Games at Recess*	6.RP.A.1	½ Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 2 Performance Task Option 1 (Optional) *Riding at a Constant Speed*	6.RP.A.2 6.RP.A.3	Teacher Discretion	Teacher Discretion	Yes, if administered

Name _____ Block _____ Date _____

Games at Recess (NJSLS 6.RP.A.1)

The students in Mr. Hill's class played games at recess.

6 boys played soccer
4 girls played soccer
2 boys jumped rope
8 girls jumped rope

Afterward, Mr. Hill asked the students to compare the boys and girls playing different games.

Mika said, "Four more girls jumped rope than played soccer."

Chaska said, "For every girl that played soccer, two girls jumped rope."

Mr. Hill said, "Mika compared the girls by looking at the difference and Chaska compared the girls using a ratio."

A) Compare the number of boys who played soccer and jumped rope using the difference. Write your answer as a sentence as Mika did.

B) Compare the number of boys who played soccer and jumped rope using a ratio. Write your answer as a sentence as Chaska did.

C) Compare the number of girls who played soccer to the number of boys who played soccer using a ratio. Write your answer as a sentence as Chaska did.

Unit 2 Performance Task 1 PLD Rubric

SOLUTION

A) Student indicates four more boys played soccer than jumped rope

B) Student indicates for every three boys that played soccer, one boy jumped rope. Therefore the ratio of the number of boys that played soccer to the number of boys that jumped rope is 3: (or "three to one").

C) Student indicates for every two girls that played soccer, three boys played soccer. Therefore the ratio of the number of girls that played soccer to the number of boys that played soccer is 2:3 (or "two to three").

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion • a logical, but incomplete, progression of steps	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • majr calculation errors • partial justification of a conclusion	No parts are correct. The student shows no work or justification.

Name _____ Block _____ Date _____

Riding at a Constant Speed (6.RP.A.2, 6.RP.A.3)

Task

Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed,

A. How far did she ride in 15 minutes?

B. How long did it take her to ride 6 miles?

C. How fast did she ride in miles per hour?

D. What was her pace in minutes per mile?

IX. 21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Illustrative Mathematics" *Open Up Resources.* 2018

<https://auth.openupresources.org/register/complete>