6th Grade Mathematics

The Number System: Division of fractions, computation of multi-digit numbers, and the system of rational numbers

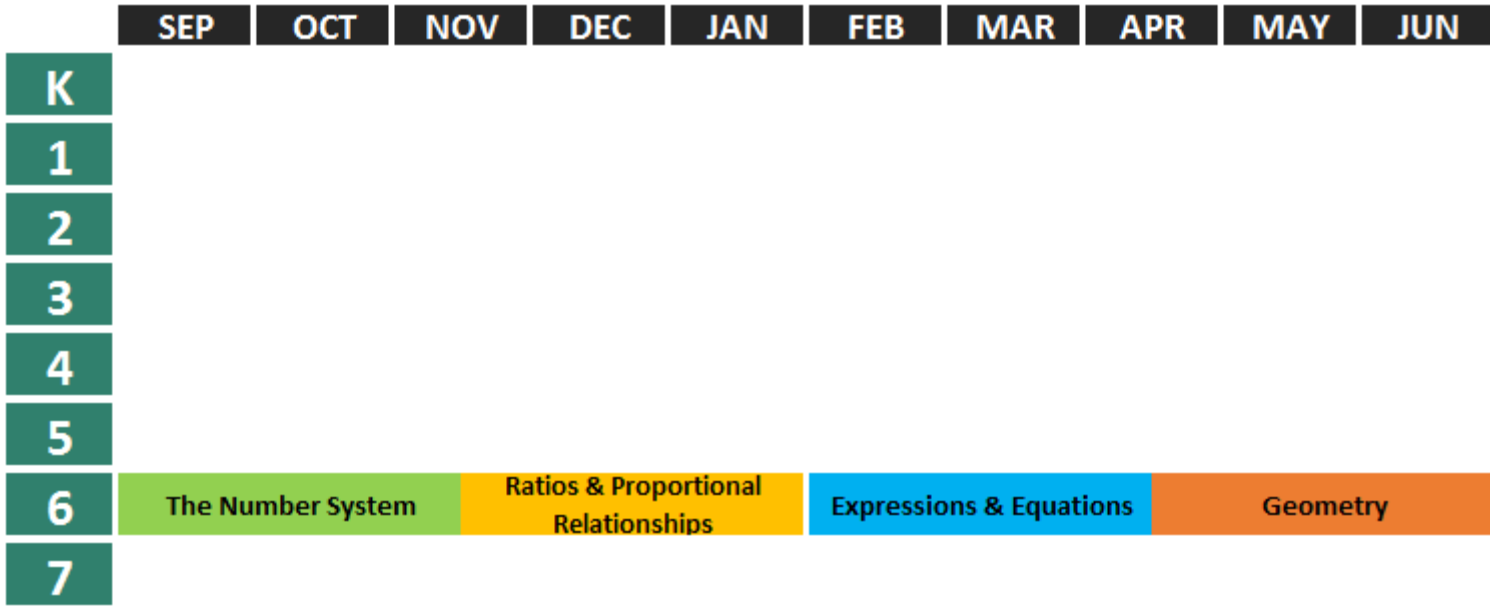
Unit 1 Curriculum Map – Math in Focus

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

A STORY OF UNITS



The Number System: Division of fractions, computation of multi-digit numbers, and the system of rational numbers

Ratios & Proportional Relationships: Understand ratio concepts and use ratio reasoning to solve problems

Expressions & Equations: Arithmetic with algebraic expressions, solve simple equations/ inequalities, and analyze relationships

Geometry: Solve real-world and mathematical problems involving area, surface area, and volume

Table of Contents

Unit Overview

In this unit students will …

- Apply knowledge of prime factorization to find the greatest common factor or least common multiple of a set of numbers.
- Square and cube to evaluate numerical expressions.
- Use absolute value to interpret real-world situations involving positive and negative numbers.
- Interpret and compute quotients of fractions.
- Solve word problems involving division of fractions by fractions using visual fraction models and equations to represent the problem.
- Fluently divide multi-digit numbers using the standard algorithm.
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Enduring Understandings

- The meanings of each operation on fractions are consistent with the meanings of the operations on whole numbers. For example: It is possible to divide fractions without multiplying by the inverse or reciprocal of the second fraction.
- When dividing by a fraction, there are two ways of thinking about the operation – partition and measurement, which will lead to two different thought processes for division.
- When we divide one number by another, we may get a quotient that is bigger than the original number, smaller than the original number or equal to the original number.

Pacing Guide

Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Chapter 1		
Chapter 1 Recall Prior Knowledge / Pre-Test (MIF)	6.NS.4, 6.EE.1, 6.EE.2, 6.NS.6, 6.NS.7a, 6.EE.2c, 8.EE.2	1 Block
Chapter 1 (MIF) Transition Lesson	6.NS.4, 6.EE.1, 6.EE.2, 6.NS.6, 6.NS.7a, 6.EE.2c, 8.EE.2	1 Block
Chapter 1 (MIF) Lesson 1.1	6.NS.6, 6.NS.7a	2 Blocks
Chapter 1 (MIF) Lesson 1.2	6.NS.4	1 Block
Chapter 1 (MIF) Lesson 1.3	6.NS.4	2 Blocks
Chapter 1 (MIF) Lesson 1.4	6.EE.1, 6.EE.2	1 Block
Chapter 1 (MIF) Lesson 1.5	6.EE.2c, 8.EE.2	2 Blocks
Chapter 1 (MIF) Wrap-Up / Review	6.NS.4, 6.EE.1, 6.EE.2, 6.NS.6, 6.NS.7a, 6.EE.2c, 8.EE.2	1 Block
Chapter 1 Assessment (MIF) *Optional*	6.NS.4, 6.EE.1, 6.EE.2, 6.NS.6, 6.NS.7a, 6.EE.2c, 8.EE.2	1 Block *Optional*
Total Time		**12 Blocks**

Major Work Supporting Content Additional Content

Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Chapter 2		
Chapter 2 Recall Prior Knowledge / Pre-Test (MIF)	6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b, 6.NS.7c, 6.NS.7d	1 Block
Chapter 2 (MIF) Transition Lesson	6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b, 6.NS.7c, 6.NS.7d	1 Block
Grade 6 Module 3 Lesson 2 (EngageNY) TE / SE	6.NS.5	1 Block
Chapter 2 (MIF) Lesson 2.1	6.NS.5, 6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b	2 Blocks
Chapter 2 (MIF) Lesson 2.2	6.NS.7c, 6.NS.7d	1 Block
Chapter 2 (MIF) Wrap-Up / Review	6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b, 6.NS.7c, 6.NS.7d	1 Block
Chapter 2 Assessment (MIF) *Optional*	6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b, 6.NS.7c, 6.NS.7d	1 Block *Optional*
Unit 1 Assessment 1	6.NS.4, 6.NS.5, 6.NS.6, 6.NS.7	1 Block
Total Time		**9 Blocks**

Major Work Supporting Content Additional Content

Activity	New Jersey Student Learning Standards (NJSLS)	Estimated Time
Chapter 3		
Chapter 3 Recall Prior Knowledge / Pre-Test (MIF)	6.NS.1, 6.NS.2, 6.NS.3	1 Block
Chapter 3 (MIF) Transition Lesson	6.NS.1, 6.NS.2, 6.NS.3	1 Block
Chapter 3 (MIF) Lesson 3.1	6.NS.1	3 Blocks
Chapter 3 (MIF) Lesson 3.2	6.NS.3	2 Blocks
Chapter 3 (MIF) Lesson 3.3	6.NS.2, 6.NS.3	2 Blocks
Chapter 3 (MIF) Lesson 3.4	6.NS.1, 6.NS.3	2 Blocks
Chapter 3 (MIF) Wrap-Up / Review	6.NS.1, 6.NS.2, 6.NS.3	2 Blocks
Chapter 3 Assessment (MIF) *Optional*	6.NS.1, 6.NS.2, 6.NS.3	1 Block *Optional*
Unit 1 Assessment 2	6.NS.1, 6.NS.2, 6.NS.3	1 Block
Total Time		**15 Blocks**

Major Work Supporting Content Additional Content

Unit 1 Overview		
Activity	**New Jersey Student Learning Standards (NJSLS)**	**Estimated Time**
Chapter 1 (MIF)	6.NS.4, 6.EE.1, 6.EE.2, 6.NS.6, 6.NS.7a, 6.EE.2c, 8.EE.2	12 Blocks
Chapter 2 (MIF)	6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b, 6.NS.7c, 6.NS.7d	9 Blocks
Chapter 3 (MIF)	6.NS.1, 6.NS.2, 6.NS.3	15 Blocks
Solidify Unit 1 Concepts / Project Based Learning		5 Blocks
Total Time		**41 Blocks**

Major Work Supporting Content Additional Content

Pacing Calendar

Please complete the pacing calendar based on the suggested pacing (*see Pacing Guide on pages 3-4)*.

SEPTEMBER						
Sunday	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
					1	**2**
3	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**
17	**18**	**19**	**20**	**21**	**22**	**23**
24	**25**	**26**	**27**	**28**	**29**	**30**

OCTOBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

NOVEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Math Background

Chapter 1: Positive Numbers and the Number Line

In this chapter, students learn that

- A single number can be represented in many ways. (Example, the number 10 can also be expressed as the fraction 10/1, the decimal 10.0, in factored form as $2 \cdot 5$, as the square root of 100, or as the cube root of 1,000.
- A number line can help students compare positive whole numbers. On a horizontal number line, numbers increase in value from left to right. On a vertical number line, numbers increase from the bottom to the top.
- The distance between any two whole numbers on a number line can be divided into fractional parts to represent and compare decimals as fractions.
- Prime numbers have exactly two factors, the number 1 and itself. The first prime number is 2 and it is the only even prime number. Similarly, 5 is the only prime ending in 5, since all whole numbers that end in 5 are divisible by 5.
- Composite numbers are greater than one and have more than two factors. Composite numbers are divisible by numbers other than themselves and 1.
- The number 1 is neither prime nor composite because it has only one factor, 1.
- Every integer greater than 1 can be expressed as a product of the prime factors in only one way (except for the order of the factors). The result will always be the same, no matter how you start to factor the number. This is called the Fundamental Theorem of Arithmetic.

Chapter 2: Negative Numbers and the Number Line

In this chapter, students learn that

- On a horizontal number line, positive numbers are to the right of 0 and negative numbers are to the left of 0.
- The number 0 is neither positive nor negative.
- Each number on the number line has an opposite, the number that is the same distance away from 0.
- Number lines help to compare and order negative and positive numbers visually.
- The (-) sign also means “negative” or “the opposite of” a number.
- Negative numbers are used to record temperatures, elevations, money owed, and other quantities.
- The absolute value of a number is its distance from 0 on a number line. Absolute value bars are used to show absolute value: I7I or I-3I.
- Since absolute value expresses the distance, the absolute value is always positive I-2I = 2
- Two opposites have the same absolute value I-4I = 4 and I4I = 4.

Chapter 3: Multiplying & Dividing Fractions and Decimals

In this chapter, students learn to

- Divide fractions
- Multiply and divide decimals.
- Relate division to multiplication.
- Apply multiplication and division skills to real-world problems involving fractions and decimals.
- Draw and revise bar models to aid solving multi-step real-world problems.

PARCC Assessment Evidence Statements

NJSLS	Evidence Statement	Clarification	Math Practices	Calculator ?
6.NS.1-2	Solve word problems involving division of fractions by fractions, For example, How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	i) Only the answer is required; explanations and representations are not assessed here. ii) Note that the italicized examples correspond to three meanings/uses of division: (1) equal sharing; (2) measurement; (3) unknown factor. These meanings/uses of division should be sampled equally. iii) Tasks may involve fractions and mixed numbers but not decimals	MP.4	No
6.NS.2	Fluently divide multi-digit numbers using the standard algorithm.	i) Tasks access fluency implicitly; simply in virtue of the fact that there are two substantial computations on the EOY (see also 6.NS.3-1, 6.NS.3-2, 6.NS.3-3, 6.NS.3-4). Tasks need not be timed. ii) The given dividend and divisor are such as to require an efficient/standard algorithm (e.g., 40584 ÷ 76). Numbers in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 40064 ÷ 16). iii) Tasks do not have a context. iv) Only the answer is required. v) Tasks have five-digit dividends and two-digit divisors, with or without remainders.	–	No
6.NS.3-1	Fluently add multi-digit decimals using the standard algorithm.	i) Tasks do not have a context. ii) Only the sum is required iii) The given addends require an efficient/standard algorithm (e.g., 72.63 + 4.875). iv) Each addend is greater than or equal to 0.001 and less than or equal to 99.999.	–	No

6.NS.3-2	Fluently subtract multi-digit decimals using the standard algorithm.	i) Tasks do not have a context. ii) Only the difference is required. iii) The given subtrahend and minuend require an efficient/standard algorithm (e.g., 177.3 – 72.635). iv) The subtrahend and minuend are each greater than or equal to 0.001 and less than or equal to 99.999. Positive differences only.	–	No
6.NS.3-3	Fluently multiply multi-digit decimals using the standard algorithm.	i) Tasks do not have a context. ii) Only the product is required. iii) The given factors require an efficient/standard algorithm (e.g., 72.3×4.8). iv) For purposes of assessment, the possibilities are 1-digit x 2-digit, 1-digit x 3- digit, 2-digit x 3-digit, 2-digit x 4-digit, or 2-digit x 5-digit.	–	No
6.NS.3-4	Fluently divide multi-digit decimals using the standard algorithm.	i) Tasks do not have a context. ii) Only the quotient is required. iii) The given dividend and divisor require an efficient/standard algorithm (e.g., $177.3 \div 0.36$). iv) Tasks are either 4-digit ÷ 2-digit or 3-digit ÷ 3-digit. (For example, $14.28 \div 0.68$ or $2.39 \div 0.684$). v) Every quotient is a whole number or a decimal terminating at the tenths, hundredths, or thousandths place.	–	No
6.NS.4-1	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.	i) Tasks do not have a context.	–	No
6.NS.4-2	Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).	i) Tasks do not have a context. ii) Tasks require writing or finding the equivalent expression with the greatest common factor.	MP.7	No

6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	i) Task do not require student to perform any computations. ii) Students may be asked to recognize the meaning of 0 in the situation, but will not be asked to explain.	MP.2	No
6.NS.6a	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.	i) Tasks have "thin context"[2] or no context.	MP.5 MP.8	No
6.NS.6b-1	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.	i) Tasks have "thin context" or no context. ii) Students need not recognize or use traditional notation for quadrants (such as I, II, III, IV). iii) Coordinates are not limited to integers.	MP.5	No

6.NS.6b-2	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. b. Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	i) Tasks have "thin context" or no context. ii) Students need not recognize or use traditional notation for quadrants (such as I, II, III, IV). iii) Coordinates are not limited to integers.	MP.5 MP.8	No
6.NS.6c-1	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram	i) Tasks have "thin context" or no context. ii) Coordinates are not limited to integers.	MP.5	No
6.NS.6c-2	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. c. Find and position pairs of integers and other rational numbers on a coordinate plane.	i) Tasks have "thin context" or no context. ii) Students need not recognize or use traditional notation for quadrants (such as I, II, III, IV). iii) Coordinates are not limited to integers.	MP.5	No
6.NS.7a	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.	i) Tasks do not have a context. ii) Tasks are not limited to integers.	MP.2 MP.5	No

6.NS.7b	Understand ordering and absolute value of rational numbers. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.	i) Tasks are not limited to integers. ii) For the explain aspect of 6.NS.7b, see 6.C.4	MP.2 MP.3 MP.5	No
6.NS.7c-1	Understand ordering and absolute value of rational numbers. c. Understand the absolute value of a rational number as its distance from 0 on the number line.	i) Tasks do not have a context. ii) Tasks are not limited to integers.	MP.2 MP.5	No
6.NS.7c-2	Understand ordering and absolute value of rational numbers. c. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.	i) Tasks have a context. ii) Tasks are not limited to integers.	MP.2	No
6.NS.7d	Understand ordering and absolute value of rational numbers. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represent a debt greater than 30 dollars.	i) Tasks may or may not contain context. ii) Tasks are not limited to integers. iii) Prompts do not present students with a number line diagram, but students may draw a number line diagram as a strategy.	MP.2 MP.5	No

Differentiated Instruction

Chapter 1

Assessment and Intervention

	ASSESSMENT	RtI STRUGGLING LEARNERS
DIAGNOSTIC	• Quick Check in Recall Prior Knowledge in Student Book A, pp. 3–4 • Chapter 8 Pre-Test in *Assessments*	• Skills 1–4 in *Transition Guide, Course 1*
ON-GOING	• Guided Practice • Lesson Check • Ticket Out the Door	• *Reteach* worksheets • *Extra Practice* worksheets • *Activity Book*, Chapter 1
END-OF-CHAPTER	• Chapter Review/Test • Chapter 1 Test in *Assessments* • ExamView® Assessment Suite CD-ROM Course 1	• *Reteach* worksheets

ELL ENGLISH LANGUAGE LEARNERS

Review the terms *factor* and *multiple*.

Say A *factor* of a whole number is a number that divides exactly into the given number with a remainder of 0.

Model Write the number 36 on the board. Then list its factor pairs: 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6. Point out that each factor of 36 is equal to or less than the number.

Say A *multiple* of a number is a number that is the product of the given number and any other whole number.

Model To show the first 5 multiples of 4, write 4 × 1 = 4, 4 × 2 = 8, 4 × 3 = 12, 4 × 4 = 16, and 4 × 5 = 20. Point out that each multiple of 4 is greater than or equal to the number.

For definitions, see Glossary, page 272, and Online Multilingual Glossary.

ADVANCED LEARNERS

- Students can explore using the Sieve of Eratosthenes, a method for finding prime numbers that was developed by the ancient Greek mathematician Eratosthenes.
- Using a hundreds grid, have students sequentially write the numbers from 1 to 100. Tell students to cross out the number 1 since it is neither prime nor composite. Students begin using the sieve by circling the next number on the grid, 2, and crossing out all multiples of 2. Students continue the process for 3, 5, and so on, until the only numbers left are prime.
- As students work, discuss with them why they can stop the process when they get to 11 *(Possible answer: The grid only goes up to 100 and 11 · 11 = 121, which is greater than 100).*

To provide additional challenges use:

- *Enrichment*, Chapter 1
- Student Book A, Brain@Work problem

Differentiated Instruction

Chapter 2

Assessment and Intervention

	ASSESSMENT	RtI STRUGGLING LEARNERS
DIAGNOSTIC	• Quick Check in Recall Prior Knowledge in Student Book A, pp. 43–44 • Chapter 2 Pre-Test in *Assessments*	• Skills 5–6 in *Transition Guide, Course 1*
ON-GOING	• Guided Practice • Lesson Check • Ticket Out the Door	• Reteach worksheets • Extra Practice worksheets • *Activity Book*, Chapter 2
END-OF-CHAPTER	• Chapter Review/Test • Chapter 2 Test in *Assessments* • ExamView® Assessment Suite CD-ROM Course 1	• Reteach worksheets

ELL ENGLISH LANGUAGE LEARNERS

Review the terms *positive* and *negative* and use a number line to indicate the position of positive and negative numbers in relation to 0.

Explain In real-world situations, you can look for clue words that tell you whether a number is positive or negative. Words such as *gain*, *above*, and *over* signal a positive number. Words such as *loss*, *below*, and *under* signal a negative number.

Model List pairs of words that indicate positive and negative values in a two-column chart. Then provide students with real-life examples and have students write positive or negative numbers to represent the situations. For example: In a football game, the home team lost 5 yards on a play. What number represents this situation? *(−5)*

For definitions, see Glossary, page 272, and Online Multilingual Glossary.

ADVANCED LEARNERS

- Students can make a time line of family events using positive and negative numbers on a number line. Explain that the numbers represent years and 0 is the year students were born. Events that happened before they were born are shown as negative numbers and events that happened after they were born are shown as positive numbers.
- Suggest that students identify 10 important events and list the numbers they will plot before they create their number lines.
- Once students have completed their time lines, have them work with partners to ask questions about the events shown. For example: What event happened the greatest number of years before your birth? After your birth?

To provide additional challenges use:

- *Enrichment*, Chapter 2
- Student Book A, Brain@Work problems

Connections to the Mathematical Practices

1	Make sense of problems and persevere in solving them
	– Make sense of real-world rate and proportion problem situations by representing the context in tactile and/or virtual manipulatives, visual, or algebraic models – Understand the problem context in order to translate them into ratios/rates
2	Reason abstractly and quantitatively
	– Understand the relationship between two quantities in order to express them mathematically – Use ratio and rate notation as well as visual models and contexts to demonstrate reasoning
3	Construct viable arguments and critique the reasoning of others
	– Construct and critique arguments regarding the proportion of a whole as represented in the context of real-world situations – Construct and critique arguments regarding appropriateness of representations given ratio and rate contexts, EX: does a tape diagram adequately represent a given ratio scenario
4	Model with mathematics
	– Model a problem situation symbolically (tables, expressions, or equations), visually (graphs or diagrams) and contextually to form real-world connections
5	Use appropriate tools strategically
	– Choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.
6	Attend to precision
	– Use and interpret mathematical language to make sense of ratios and rate – Attend to the language of problems to determine appropriate representations and operations for solving real-world problems. – Attend to the precision of correct decimal placement used in real-world problems
7	Look for and make use of structure
	– Use knowledge of problem solving structures to make sense of real world problems – Recognize patterns that exist in ratio tables, including both the additive and multiplicative properties – Use knowledge of the structures of word problems to make sense of real-world problems

8	**Look for and express regularity in repeated reasoning** – Utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts – Generalize the relationship between representations, understanding that all formats represent the same ratio or rate – Demonstrate repeated reasoning when dividing fractions by fractions and connect the inverse relationship to multiplication – Use repeated reasoning when solving real-world problems using rational numbers

Vocabulary

Term	Definition
Chapter 1 Vocabulary	
Base (of an exponent)	In an expression of the form a^n, the base *a* is used as a factor *n* times:
Common Factor	A number that is a factor of two or more whole numbers.
Common Multiple	A number that is a multiple of two or more whole numbers.
Composite Number	A counting number that has more than two factors.
Cube (of a number)	The value of the number raised to an exponent of 3.
Cube Root	A number which, when cubed, is equal to a given number
Exponent	The number to which the base is raised. Example: In 4^5, the exponent is 5.
Multiple	The product of a whole number and any whole number. Example: 16 is a multiple of 4.
Number Line	A horizontal or vertical line representing whole numbers, fractions, and decimals.
Numerical Expression	A collection of numbers and operations symbols that represent a single value. Example: 3 x 2 + 7
Perfect Cube	The cube of a whole number.
Perfect Square	The square of a whole number.
Positive Number	A number that is greater than zero.
Prime Factor	A factor of a number that is also a prime number.
Prime Number	A counting number that has exactly two different factors, 1 and itself. Example: 5 is a prime number because its only factors are 1 and 5.
Square (of a number)	The value of the number raised to an exponent of 2.
Square Root	A number which, when squared, is equal to a given number.
Whole Number	Any of the numbers 0, 1, 2, 3, 4, and so on.
Chapter 2 Vocabulary	
Absolute Value	The distance of a number from zero on a number line.
Negative Number	A number that is less than zero.
Opposite	Having the same numerals but different signs.

Chapter 3 Vocabulary	
Improper Fraction	A fraction in which the numerator is greater than or equal to the denominator.
Mixed Number	A number with a whole number part and a fraction part.
Reciprocals	Two numbers whose product is 1.

Potential Student Misconceptions

- Students may believe that dividing by $1/2$ is the same as dividing in half. Dividing by half means to find how many one-halves there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts. $7 \div 1/2 = 14$ and $7 \div 1/2 \neq 3$.

- Students may understand that $1\frac{1}{2} \div \frac{1}{4}$ means, "How many fourths are in $1\frac{1}{2}$?" So, they set out to count how many fourths (6). But in recording their answer, they can get confused as to what the 6 refers to and think it should be a fraction, and they record 6/4 when actually it is 6 groups of one-fourths, not 6 fourths

- As noted above, knowing what the unit is (the divisor) is critical and must be understood in giving the remainder. In the problem 3 3/8 ÷ 1/4, students are likely to count 4 fourths for each whole number (12 fourths) and one more for 2/8, but then not know what to do with the extra eights. It is important to be sure they understand the measurement concept of division. Ask, "How much of the next piece do you have?" Context can also help. In this case, if the problem was about pizza servings, there would be 13 full servings and $1/2$ of the next serving.

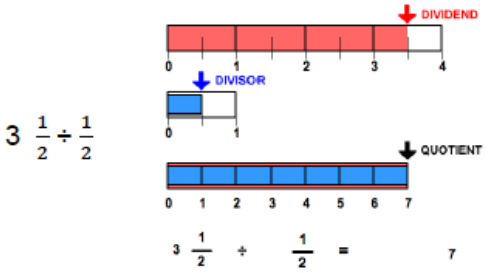
- The most common error in adding fractions is to add both the numerators and the denominators. For example, one teacher asked her fifth graders if the following was correct: 3/8 + 2/8 = 5/16. A student correctly replied, "No because they are eighths. If you put them together, you will still have eighths. See, you can't make them into sixteenths when you put them together. They are still eighths."

- Many students have trouble finding common denominators because they are not able to come up with common multiples of the denominators quickly. This skill requires having a good command of multiplication facts. Students benefit from knowing that any common denominator will work. Least common denominators are preferred because the computation is more manageable with smaller numbers, and there is less simplifying to do after adding or subtracting. Do not require least common multiples, support all common denominators, and through discussion students will see that finding the smallest multiple is more efficient.

Teaching Multiple Representations

CONCRETE REPRESENTATIONS

- Number Lines



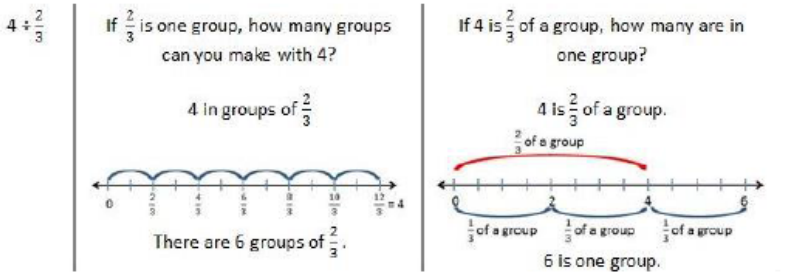
- 2-color coin counters to represent negatives and positives

- Number Lines

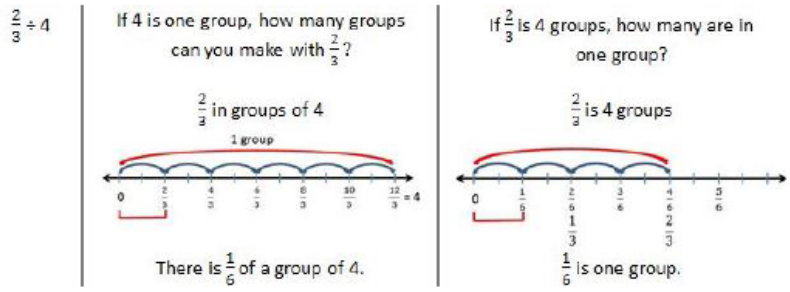
- Thermometers and other equally partitioned tools

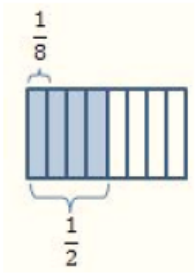
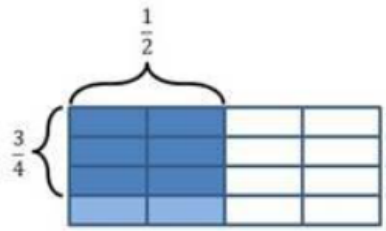
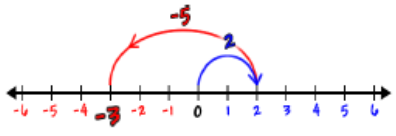
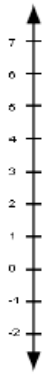
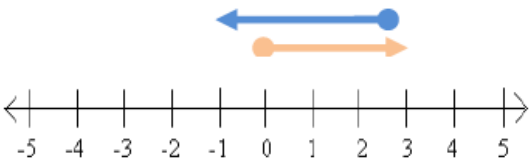
PICTORIAL REPRESENTATIONS

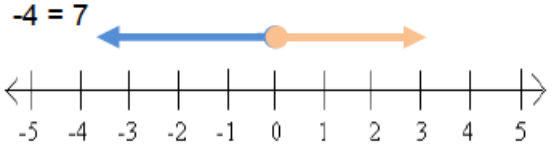
- Number Lines (Division Shown)



- Rectangular Area Models (Division & Multiplication Shown)



	$\frac{1}{2} \div \frac{1}{8}$  $\frac{3}{4} \times \frac{1}{2}$ 
• Number Lines (Horizontal) • Number Lines (Vertical)	 Figure 3 - Vertical Number Line 
• Distance / Vector Model	*Adding Integers* Addition is modeled as putting a second vector's tail at the first vector's head and finding where the second vector's head extends to. 3 + -4 = -1 

	Subtracting Integers Subtraction can be thought of as comparing the two vectors p, and q, by putting both tails together (starting each from zero) and asking the question: "How would one extend a vector from the head of p to the head of q?" The length and direction of that vector would be the result of the subtraction. 3 - -4 = 7 
ABSTRACT REPRESENTATIONS	
• Applying the Operations • Applying Properties of Numbers • Applying the standard algorithms for addition, subtraction, multiplication, and division	
• Applying Properties of Numbers	p – q = p + (-q) p - -q = p + q

Assessment Framework

Unit 1 Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Chapter 1 Pretest (Beginning of Unit) *Math in Focus*	6.NS.4, 6.EE.1, 6.EE.2, 6.NS.6, 6.NS.7a 6.EE.2c, 8.EE.2	½ Block	Individual	Yes (No Weight)
Chapter 2 Pretest (After Chapter 1) *Math in Focus*	6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b, 6.NS.7c, 6.NS.7d	½ Block	Individual	Yes (No Weight)
Unit 1 Assessment 1 (After Chapter 2) *District Assessment*	6.NS.4, 6.NS.6, 6.NS.7	½ Block	Individual	Yes
Chapter 3 Pretest (After Unit 1 Assessment 1) *Math in Focus*	6.NS.1, 6.NS.2, 6.NS.3	½ Block	Individual	Yes (No Weight)
Unit 1 Assessment 2 (Conclusion of Unit) *District Assessment*	6.NS.1, 6.NS.2, 6.NS.3	½ Block	Individual	Yes
Chapter 1 Test (Optional) *Math in Focus*	6.NS.4, 6.EE.1, 6.EE.2, 6.NS.6, 6.NS.7a 6.EE.2c, 8.EE.2	½ Block	Individual	Yes, if administered
Chapter 2 Test (Optional) *Math in Focus*	6.NS.6, 6.NS.6a, 6.NS.7a, 6.NS.7b, 6.NS.7c, 6.NS.7d	½ Block	Individual	Yes, if administered
Chapter 3 Test (Optional) *Math in Focus*	6.NS.1, 6.NS.2, 6.NS.3	½ Block	Individual	Yes, if administered

Unit 1 Performance Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded ?**
Unit 1 Performance Task 1 (Late September) *Extending the Number Line*	6.NS.C.6	1 Block	Individual	Yes; Rubric
Unit 1 Performance Task 2 (Mid October) *Jumping Flea*	6.NS.C.7	1 Block	Individual w/ Interview Opportunity	Yes; Rubric
Unit 1 Performance Task 3 (Late October) *How Many Containers in One Cup/ Cups in One Container?*	6.NS.A.1	1 Block	Individual	Yes; Rubric
Unit 1 Performance Task Option 1 (Optional)	6.NS.C.7a	Teacher Discretion	Teacher Discretion	Yes, if administered

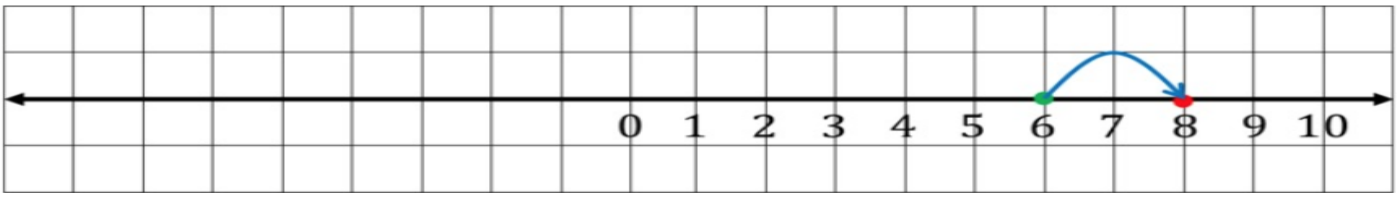
Performance Tasks

Unit 1 Performance Task 1

Extending the Number Line (6.NS.c.6)

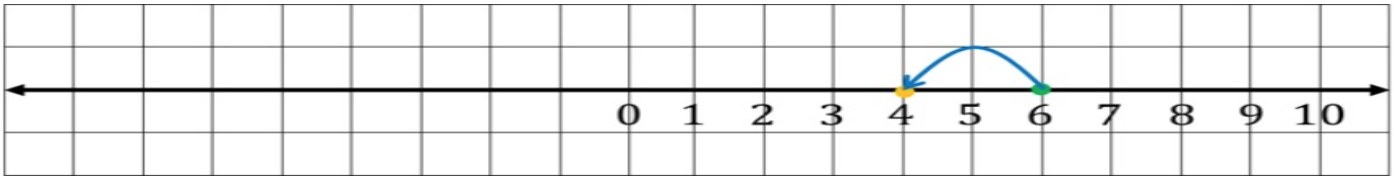
Task

A. Draw a line on graph paper. Make a tick mark in the middle of the line and label it 0. Mark and label 1, 2, 3, ... 10. Since 6+2 is 2 units to the right of 6 on the number line, we can represent 6+2 like this:



Describe the location of 3+4 on the number line in terms of 3 and 4. Draw a picture like the one above.

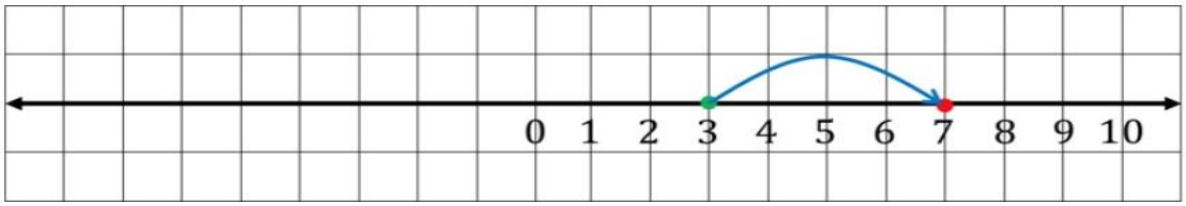
B. 6-2 is 2 units to the left of 6 on the number line, which we can represent like this:



Describe the location of 3-4 on the number line in terms of 3 and 4. Draw a picture like the one above

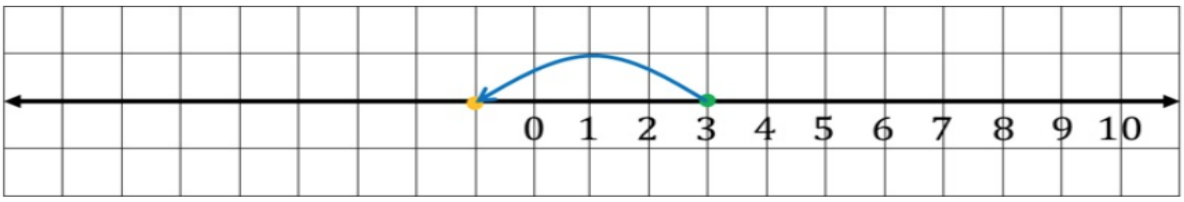
Solution

A. Since 3+4 is 4 units to the right of 3 on the number line, we can find and represent 3+4 like this:



We can see from the number line that 3+4=7.

B. Since 3-4 is 4 units to the left of 3 on the number line, we can find and represent 3-4 like this:



The difference 3-4 has a well determined place on the number line as shown in the picture. This place, however, is not marked and is not a whole number. The difference 3-4 is one unit to the left of 0.

Unit 1 Performance Task 1 PLD Rubric

SOLUTION

A) Student indicates & illustrates 3+4 is 4 units to the right of 3 on the number line.

B) Student indicates & illustrates the difference 3-4 is one unit to the left of 0.

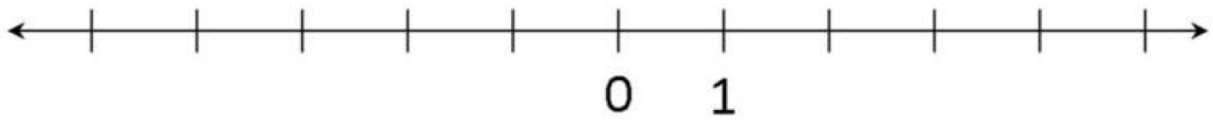
Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusion with minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusionwith minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

Unit 1 Performance Task 2

Jumping Flea (6.NS.c.7)

Task

A flea is jumping around on the number line.



a. If he starts at 1 and jumps 3 units to the right, then where is he on the number line? How far away from zero is he?

b. If he starts at 1 and jumps 3 units to the left, then where is he on the number line? How far away from zero is he?

c. If the flea starts at 0 and jumps 5 units away, where might he have landed? d. If the flea jumps 2 units and lands at zero, where might he have started?

e. The absolute value of a number is the distance it is from zero. The absolute value of the flea's location is 4 and he is to the left of zero. Where is he on the number line?

Solution

It would be a good idea to use a number line to illustrate these solutions.

a. If he starts at 1 and jumps 3 units to the right, then the flea is at 4. He is 4 units away from zero.

b. If he starts at 1 and jumps 3 units to the left, then the flea is at -2. He is $|-2| = 2$ units away from zero.

c. If the flea starts at 0 and jumps five units away, then he is either at -5 or 5.

d. If the flea lands on 0 and jumped 2 units, then he started at either -2 or 2.

e. If the absolute value of the flea's location is 4, then he is either at -4 or 4. Since he is to the left of zero, the flea is at -4.

Unit 1 Performance Task 2 PLD Rubric

SOLUTION

- A) Student indicates 4. (The flea is 4 units away from zero).
- B) Student indicates -2. (The flea is 2 units away from zero).
- C) Student indicates 5 or -5. (The flea is 5 units away from zero).
- D) Student indicates 2 or -2. (The flea is 2 units away from zero).
- E) Student indicates -4. (The flea is 4 units to the left of zero).

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusionwith minor computational error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical approach based on a conjecture and/or stated assumptions • a logical and complete progression of steps • complete justification of a conclusionwith minor conceptual error	Clearly constructs and communicates a complete response based on concrete referents provided in the prompt or constructed by the student such as diagrams that are connected to a written (symbolic) method, number line diagrams or coordinate plane diagrams, including: • a logical, but incomplete, progression of steps • minor calculation errors • partial justification of a conclusion	Constructs and communicates an incomplete response based on concrete referents provided in the prompt such as: diagrams, number line diagrams or coordinate plane diagrams, which may include: • a faulty approach based on a conjecture and/or stated assumptions • An illogical and Incomplete progression of steps • major calculation errors • partial justification of a conclusion	The student shows no work or justification.

Unit 1 Performance Task 3

How Many Containers in One Cup / Cups in One Container? (6.NS.A.1)

a. If 1/2 cup of water fills 2/3 of a plastic container, how many containers will 1 cup fill?
   - Solve the problem by drawing a picture.
   - Which of the following multiplication or divisions problems represents this situation? Explain your reasoning.

     $1/2 \times 2/3 = ?$ $1/2 \div 2/3 = ?$ $2/3 \div 1/2 = ?$
   - Solve the arithmetic problem you chose in part (3) and verify that you get the same answer as you did with your picture.

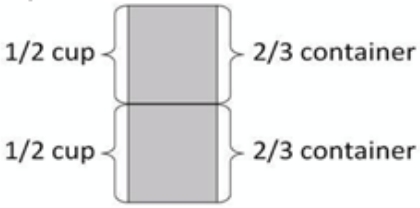
b. If 1/2 cup of water fills 2/3 of a plastic container, how many cups of water will the full container hold?
   - Solve the problem by drawing a picture.
   - Which of the following multiplication or divisions equations represents this situation? Explain your reasoning.

     $1/2 \times 2/3 = ?$ $1/2 \div 2/3 = ?$ $2/3 \div 1/2 = ?$
   - Solve the arithmetic problem you chose in part (3) and verify that you get the same answer as you did with your picture.

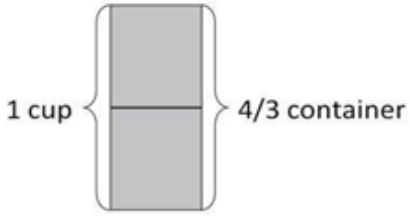
Solution

Part A: Below is a series of pictures that can be used to solve the problem as well as an explanation of what the picture represents at each step.
1/2cup of water fills 2/3 of a plastic container, as shown below.

We know that 1/2 cup of water fills 2/3 of a plastic container. The question is asking, "How many containers can be filled with 1 cup of water." Since 1 cup of water is twice as much as 12 cup of water, we know that 1 cup of water will fill twice as much of a container.



The answer to the question can easily be found by relabeling the above picture.



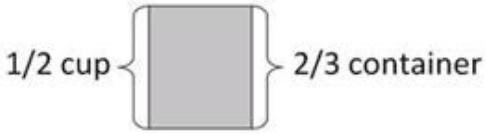
Twice as much water will fill twice as many containers. Thus, 1 cup of water will fill 4/3 of a container. We are looking for the number of containers filled by 1 cup, and we know 12 of that unknown amount fills 2/3 of a container. We can write this symbolically as
$1/2 \times ? = 2/3$ Which is equivalent to $2/3 \div 1/2$ Computing this by multiplying by the reciprocal, we find that

$$2/3 \div 12 = 2/3 \times 2/1 = 4/3$$

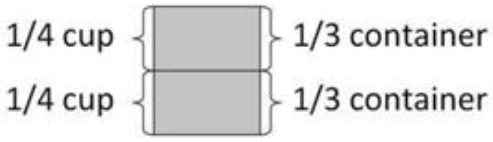
When we solved the problem with a picture, we took the fraction of the container filled by 1/2 cup and doubled it to find the fraction of a container filled by one cup. This is exactly what we did when we did it through computations: to divide 2/3 by 1/2 we multiplied it by 2. The two ways of solving it do result in the same answer and even follow the same process.

Part B: Below is a series of pictures that can be used to solve the problem as well as an explanation of what the picture represents at each step.

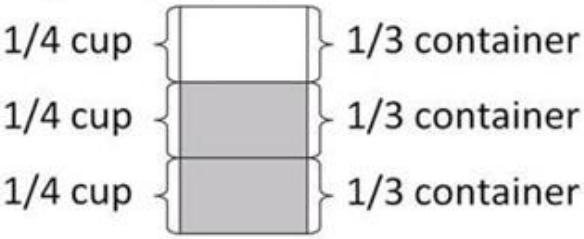
1/2 cup fills 2/3 of the container, as shown in the picture below.



Since the picture represents 2/3 of the container, 1/2 of 2/3 will be 1/3 of the container. In the picture below, each piece represents 1/2 of 2/3, and also represents 1/2 of 1/4. Since 12×1/2=1/4, each piece also represents 1/4 cup.



We have 2/3 of the container, but we need to know how many cups it takes to fill 3/3 of the container, so if we take 3 of the thirds, we will have the amount of water needed for the full container. This is the same thing as multiplying 3×1/4.



We can see that 3/4 cup is needed to fill the whole container.
We are looking for the number of cups needed to fill one container, and we know 2/3 of that unknown amount requires 1/2 a cup. We can write this symbolically as

$$2/3\times ?=1/2$$

which is equivalent to 1/2÷2/3 Computing this by multiplying by the reciprocal, we find that

$$1/2\div 2/3=1/2\times 3/2=3/4$$

When we solved the problem using a picture, we found 12 of the amount (because we knew it was 2 thirds of the container, and so 1/2 of that would be 1 third of the container) and then we multiplied that amount by 3 to get the amount in the whole container. This is exactly what we did when we multiplied by the reciprocal of 2/3: multiplying by 1/2 and then by 3 is the same as multiplying by 3/2. So whether we solve it with a picture or via computation, we get the same answer (in fact, the process itself is the same).

Unit 1 Performance Task 3 PLD Rubric

SOLUTION

- Student indicates 1 cup of water will fill 4/3 of a container, states that it's a division problem and shows both the picture and arithmetic work.
- Student indicates 3⁄4 cup of water will fill up a container, states that it's a division problem and shows both the picture and arithmetic work.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
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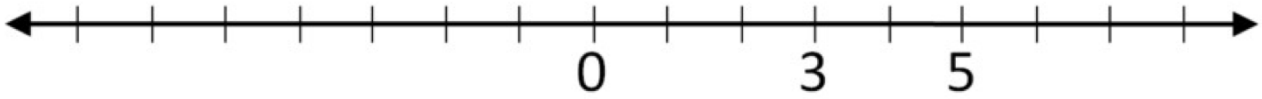
Unit 1 Performance Task Option 1

Integers on the Number Line (6.NS.C.7a)

Task



a. Find and label the numbers −3 and −5 on the number line.

b. For each of the following, state whether the inequality is true or false. Use the number line diagram to help explain your answers.

i. −3 > −5

ii. ii. −5 > −3

iii. iii. −5 < −3

iv. iv. −3 < −5

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

Extensions and Sources

Online Resources

http://www.illustrativemathematics.org/standards/k8
- Performance tasks, scoring guides

http://www.ixl.com/math/grade-6
- Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/math/arithmetic/fractions
- Interactive, tracks student points, objective descriptive videos, allows for hints

https://www.khanacademy.org/math/arithmetic/rates-and-ratios
- Interactive, tracks student points, objective descriptive videos, allows for hints

http://www.doe.k12.de.us/assessment/files/Math_Grade_6.pdf
- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx
- Common core assessments and tasks designed for students with special needs

http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsGRADE8_Nov2012V3_FINAL.pdf
- PARCC Model Content Frameworks Grade 8