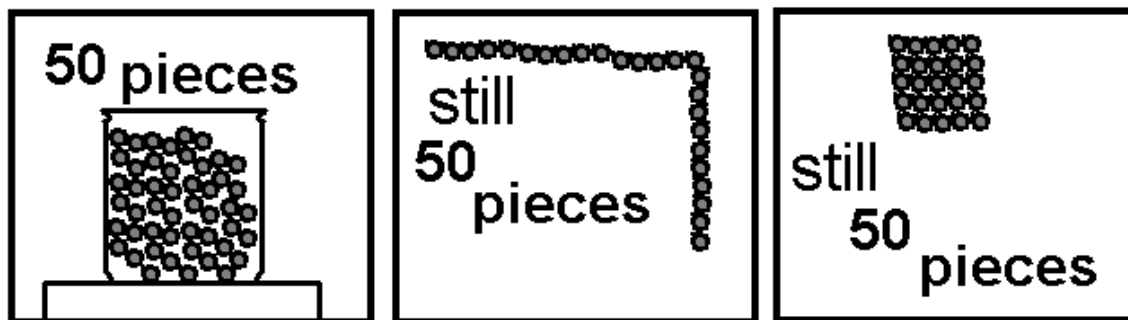


Energy

Introduction

Some of the most powerful tools in physics are based on conservation principles. The idea behind a conservation principle is that there are some properties of systems that don't change, even though other things about the system may.

For instance, let's say that I have a package of candy that contains exactly 50 pieces. If I take those pieces of candy out of the package and put them on top of a table, I still have 50 pieces. If I lay them end-to-end or arrange them into a rectangle, I still have 50 pieces. No matter how many different ways I arrange them; I still have 50 pieces. They may look different in each case, but the total number stays the same. In this example, I could say that the number of pieces of candy is conserved.



Energy is another example of a conserved property of a system. It's hard to come up with a meaningful definition of energy. It's a basic property of the universe (like time and space) so it's very hard to define. It's a lot easier to visualize a piece of candy than a piece of energy. However, it is possible to mathematically describe the various forms of energy. Having done that, it has been consistently proven true that if you add up all the types and amounts of energy within a closed system the total amount of energy does not change.

To work with this definition it's important to understand the idea of a closed system. It is true that I could change the number of pieces of candy on the table by eating some of them, dropping a piece on the floor or opening another package and spilling some of that new candy onto the table. We have to account for any candy that's been added to or taken away from the amount that we started with or we'll see that the number has changed and our conservation principle will seem to have been violated.

The same thing is true for energy. The amount of energy in a closed system stays constant. But this means that if we add or take away energy from a system, we have to account for it. Unless we do that successfully, it will appear that the Conservation of Energy principle has been violated. One way that we can move energy into or out of a system is called Work. Work has a very specific mathematical definition in physics and it represents the movement of mechanical energy into or out of a system. If work is the only means to move energy into or out of our system then it will be true that

Initial Amount of Energy + Work = Final Amount of Energy

Or

$$E_0 + W = E_f$$

The forms that energy takes can vary quite widely. Some of these forms include gravitational, electrical, chemical, kinetic, magnetic, elastic and nuclear. These are just some of the forms that energy may take, but

there are many more. In this chapter, we'll be discussing several mechanical forms of energy, kinetic energy, gravitational potential energy and elastic potential energy, along with the concept of work.

Work

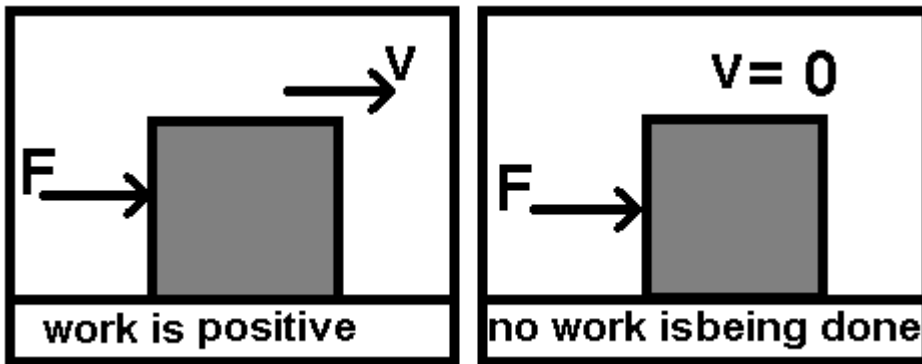
Work is defined as the product of the force applied to an object and the distance that the object moves in the direction of that force. The mathematical description of that definition is:

$$\text{Work} = \text{Force} \times \text{Distance}_{\text{parallel}}$$

Or

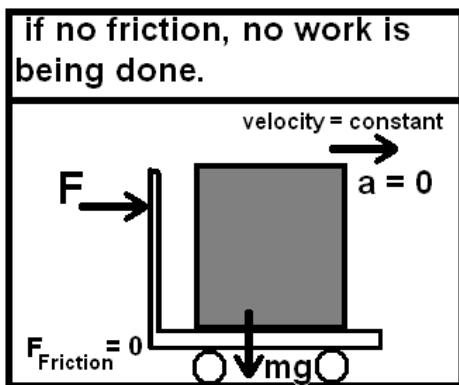
$$W = F d_{\text{parallel}}$$

It is important to note that work is proportional to the product of the force and the distance that the object moves parallel to that force. That means that if the object moves in the direction that I am pushing or pulling it, then I am doing work. If it does not move, or if it moves perpendicular to the direction that I am pushing or pulling it, I am not doing any work.

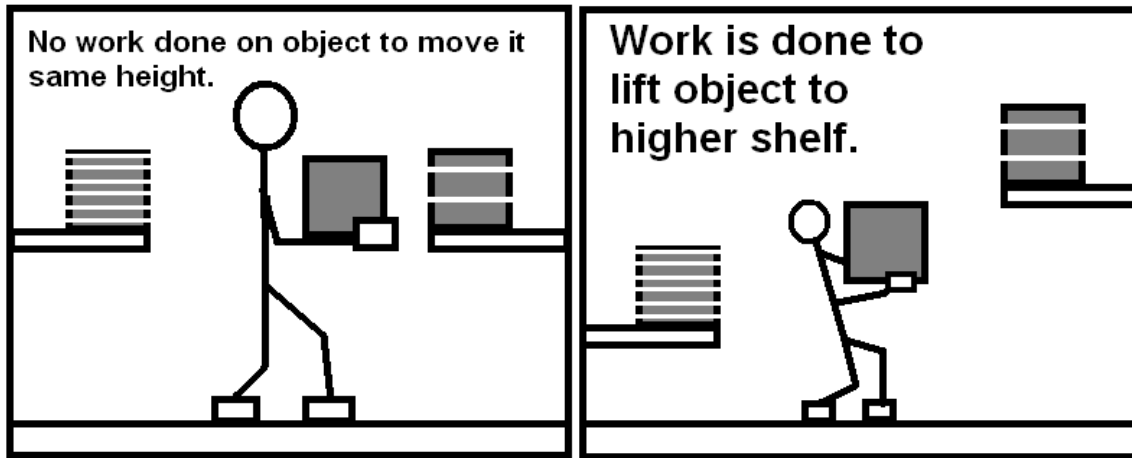


This can be confusing because the use of the word “work” in English is similar to but not the same as its use in physics. For instance if someone were to pay me to hold a heavy box up in the air while they move a table to sweep underneath it, I would say that I am doing work. But I would not be doing work according to the physics definition of the term. That is because the box is not moving in the direction of the force that I am applying. I am applying a force upwards but the box is stationary. Since d_{parallel} is equal to zero, so is the amount of work, W .

The same thing applies if I were to put that heavy box on a perfectly frictionless cart and push it to the side of the room at a constant velocity. Since the velocity is constant, the acceleration is zero. If there's no friction to overcome, then the force I need to apply (once I've gotten it moving) is zero. No force... no work.



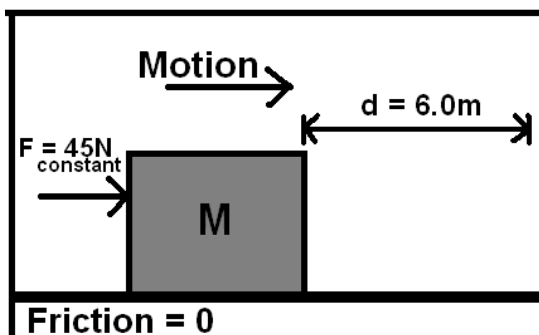
The last way that I cannot do work (according to the physics definition of work) is if I were to carry a box across the room at a constant velocity and put it on a shelf at the same height. Once again, I don't need to apply a horizontal force to keep moving at a constant velocity so there are no forces in the horizontal direction. I am applying a force in the vertical direction, to keep the box from falling to the ground. But the box is not moving in the vertical direction, it's moving in the horizontal direction. So in the horizontal direction, the force is equal to zero and in the vertical direction d_{parallel} equals zero. The result is that $W = 0$ in both cases.



While our definition of work may not always seem to relate to our experience, it turns out to be a very useful tool in developing a theory of energy. In fact, the three forms of energy that we will be discussing in this chapter all become clear through thinking about them with respect to work.

Units of Energy
 The unit of energy can be derived from the basic equation of work.
 $W = F \times d_{\text{parallel}}$
 The SI units of force are Newtons (N) and of distance are meters (m). Therefore, the units of energy are Newton-meters (N-m). Out of respect for James Prescott Joule (1818-1889), a key formulator of the concept of energy, this is also referred to as a Joule (J).
 $J = \text{N}\cdot\text{m}$
 $J = (\text{kg}\cdot\text{m} / \text{s}^2) \cdot \text{m}$
 $J = \text{N}\cdot\text{m} = \text{kg}\cdot\text{m}^2 / \text{s}^2$

Example 1: A constant force of 45 N is applied to a mass on a frictionless surface. The force is applied in the same direction as the motion of the object. How much work does that force do over a distance of 6.0m?



Since the force and the distance that the object moves are parallel to one another the work done by the force will simply be the product of the two.

$$\begin{aligned}
 W &= F d_{\text{parallel}} \\
 &= 45 \text{ N} \times 6\text{m} \\
 &= 270 \text{ N}\cdot\text{m} \\
 &= 270 \text{ J}
 \end{aligned}$$

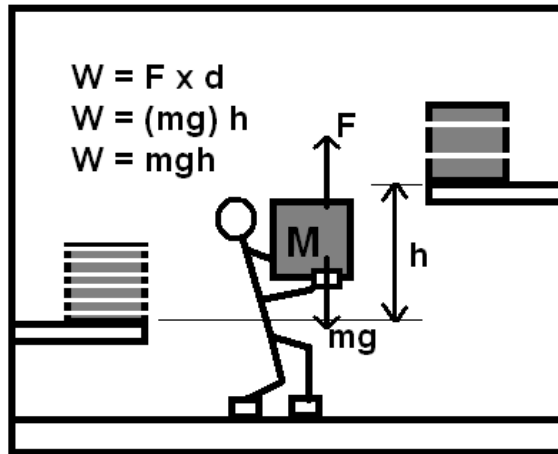
Example 2: A force of 45N keeps an object moving in circular motion at a constant speed on a horizontal frictionless surface. The circumference of the circle is 6.0m. How much work does that force do during one rotation?

The force needed to keep an object moving in a circle at a constant speed on a horizontal frictionless surface is directed towards the center of the circle. However, the velocity of the object is always tangent to the circle. Therefore F and d are always perpendicular to one another. As a result, both d_{parallel} and the work done by the force are equal to zero.

Gravitation Potential Energy

Imagine lifting a box off the floor of your room and putting it on a shelf. The shelf is at a height “ h ” and the box has a mass “ m ”. Lifting the box off the floor requires you to supply a force at least equal to its weight, “ mg ”. That means that you have to do some work, since you are lifting the box in the same direction that it moves (straight up). In fact, at the beginning you had to supply a little more than that amount of force to get it moving up and at the end a little less than that to get it to slow down to a stop, but on average the force applied would exactly equal mg .

$$\begin{aligned}
 W &= F \times d_{\text{parallel}} \\
 W &= (mg) h \\
 W &= mgh
 \end{aligned}$$



But our definition of conservation of energy tells us that

$$E_0 + W = E_f$$

That means that the work you just did must have added energy to the system in the amount of

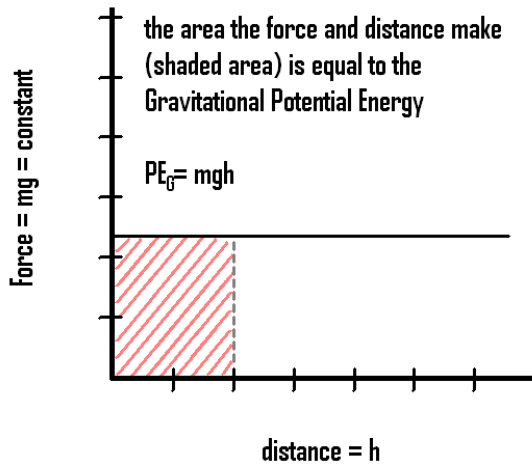
$$W = E_f - E_0$$

Since the work that you did was equal to mgh that means that the energy of the system must have been increased by that amount

$$E_f - E_0 = mgh$$

This extra energy must be stored in some form. This leads us to our first type of energy, Gravitation Potential Energy (GPE). The term GPE reflects the fact that this energy is due to the change of height of a mass located in the earth's gravitational field. The Gravitational Potential Energy, GPE, of a system is given by:

$$GPE = mgh$$

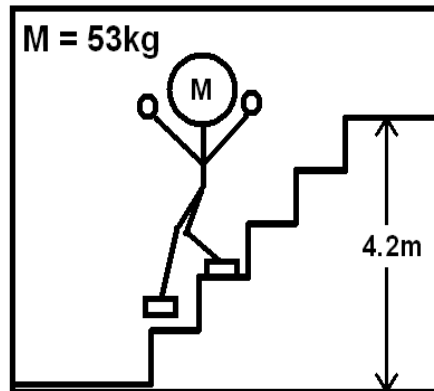


Example 3: Determine the GPE (relative to the floor) of a 50 kg box located 12m above the floor.

$$\begin{aligned} GPE &= mgh \\ &= (50\text{kg})(9.8 \text{ m/s}^2)(12\text{m}) \\ &= 5880 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\ &= 5900 \text{ J} \end{aligned}$$

Example 4: A 52 kg man walks down a 4.2m tall flight of stairs. How much work did he do?

$$\begin{aligned} E_0 + W &= E_f \\ W &= E_f - E_0 \\ &= 0 - GPE_0 \\ &= -mgh_0 \\ &= -(52\text{kg})(9.8 \text{ m/s}^2)(4.2\text{m}) \\ &= -2140 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\ W &= -2100 \text{ J} \end{aligned}$$



Kinetic Energy

Remember the box that you put up on the shelf a few pages ago. Well, simply knocking that box off the shelf will allow us to derive our second form of energy. The box will fall towards the floor with an acceleration equal to g . We can determine its velocity just before impact by using our third kinematics equation.

$$\begin{aligned} v^2 &= v_0^2 + 2ad \\ v^2 &= 0 + 2gh \\ v^2 &= 2gh \\ gh &= v^2 / 2 \end{aligned}$$

The box is now just above the floor and moving with the velocity indicated above. No energy moved into or out of the system during this process. That means the energy is the same as it was when the box was on the shelf, mgh . Therefore

$$E_0 + W = E_f$$

$$E_0 + 0 = E_f$$

$$mgh = E_f$$

$$gh = E_f / m$$

We can now set this equal to the expression for “gh” that we derived above

$$v^2/2 = E_f / m$$

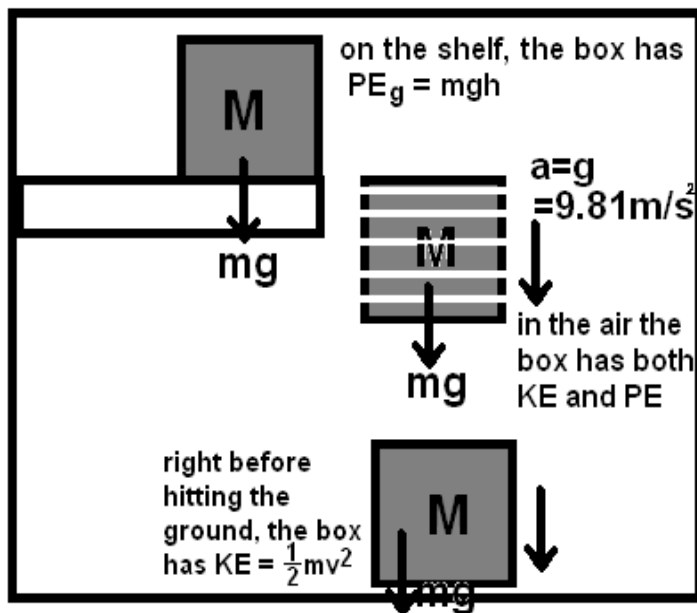
Or

$$E_f = \frac{1}{2} mv^2$$

Just before the mass strikes the ground its GPE (relative to the ground) must equal zero since its height is zero. The box still has the same amount of energy but it's in a new form since it is no longer in the form of GPE. In this case, the energy is stored in the form of a mass moving with a velocity. That form of mechanical energy is called Kinetic Energy and is given by

$$KE = \frac{1}{2} mv^2$$

In knocking the box off the shelf we started a process that converted its Gravitational Potential Energy into Kinetic Energy. It went from being a motionless mass at a height to a moving mass near the floor.



We can't follow our box as it strikes the floor, as that would introduce a whole new set of complicated energies. The reason it was so difficult to develop a good energy theory was these very complexities. We would have to account for the energy that gets transferred into the motion of the atoms that comprise the box and the floor (heat), the crashing sound of the box hitting the floor (sound energy) and the permanent deformation of the box and the floor.

It is possible to account for these in a more detailed study, but that is not necessary for what we need to do in this course. It is worth pointing out however, that the experiments of Joule proved decisive in putting the energy theory on solid ground. Joule was able to show that thermal energy was able to account for the “missing energy” in many examples like the one that we have been discussing. By doing this, the way was cleared to develop a complete and powerful theory. For this reason, the unit of energy used in the SI system is called the Joule.

Example 5: Determine the KE of a 3.0 kg ball which has a velocity of 2.1 m/s.

$$\begin{aligned}
 KE &= \frac{1}{2} mv^2 \\
 &= \frac{1}{2} (3.0\text{kg})(2.1\text{m/s})^2 \\
 &= (1.5 \text{ kg})(4.41\text{m}^2/\text{s}^2) \\
 &= 6.615 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\
 &= 6.6 \text{ J}
 \end{aligned}$$

Example 6: Use conservation of energy to determine how high a ball will go if it leaves the ground with a velocity of 24 m/s.

$$E_0 + W = E_f$$

but $W = 0$ so

$$E_0 = E_f$$

the energy is either GPE or KE so

$$KE_0 + GPE_0 = KE_f + GPE_f$$

then substitute in the formulas for each

$$\frac{1}{2} mv_0^2 + mgh_0 = \frac{1}{2} m(v_f)^2 + mgh_f$$

but h_0 and v_f are both = 0 so

$$\frac{1}{2} mv_0^2 = mgh_f$$

divide both sides by m to cancel it out

$$\frac{1}{2} v_0^2 = gh_f$$

double both sides

$$v_0^2 = 2gh_f$$

divide both sides by g and 2

$$h_f = v_0^2 / 2g$$

substitute in the given values

$$h_f = (24 \text{ m/s})^2 / 2(9.8 \text{ m/s}^2)$$

$$h_f = (576 \text{ m}^2/\text{s}^2) / 2(9.8 \text{ m/s}^2)$$

$$h_f = 29.4 \text{ m}$$

It's important to note that the height to which an object will rise does not depend on its mass. It only depends on its initial velocity.

Elastic Potential Energy

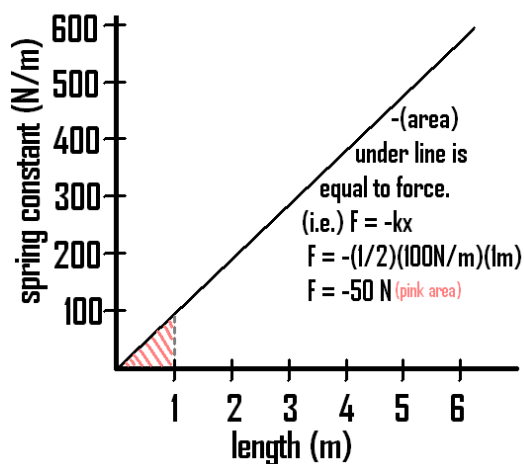
The final form of energy that we will be discussing in this chapter is Elastic Potential Energy. It represents the energy that can be stored in a spring. First, we must understand the force that is required to compress or stretch springs. This was first explained by Robert Hooke and is therefore referred to as Hooke's law.

Hooke observed that it takes very little force to stretch a spring a very small amount. However, the further the spring it stretched, the harder it is to stretch it further. The force needed increases in proportion to the amount that it has already been stretched. The same was observed to be true when compressing a spring. This can be stated mathematically as

$$F_{\text{spring}} = -k x$$

In this equation, k represents the spring constant (a characteristic of the individual spring), and x represents the distance the spring is stretched or compressed from its natural length. The negative sign tell us that the force that the spring exerts is back towards its equilibrium length, its length when it is not being stretched or compresses.

Therefore, if the spring constant for a particular spring were 100 N/m, I would need to exert a force of 100 Newtons to stretch, or compress, it by a length of 1m. If I were to exert a force of 50N, it would stretch 1/2 m. A force of 10 N would stretch, or compress, it by a distance of 1/10 m. This is shown graphically below.



It is worthwhile to compare the above graph to a graph of you lifting the box onto the shelf in our discussion of gravitational potential energy. In that case, the force was constant, mg , regardless of the distance that the box was raised. The graph of that is shown below. Also shown on that graph is the fact that the area of the rectangle that is formed by the force, mg , on the y-axis and the distance, h , on the x-axis, is equal to mgh . This is equal to the work needed to lift the box to the height of the shelf (and also to the gravitational potential energy it had upon attaining that height). Thus the work needed to lift the box is equal to the area under the Force versus Distance curve.

Similarly, in the case of a spring, the work needed to stretch or compress it will be equal to the area under its force versus distance curve. However, as can be seen below, in this case the shape that is formed is a triangle with a height equal to kx (equal and opposite to the force exerted by the spring, $-kx$) and a base equal to x . The area of a triangle is given by $\frac{1}{2}$ base times height. Therefore the work needed to compress or stretch a spring a distance x is given by

$$W = \text{Area under the } F \text{ vs. } d \text{ curve}$$

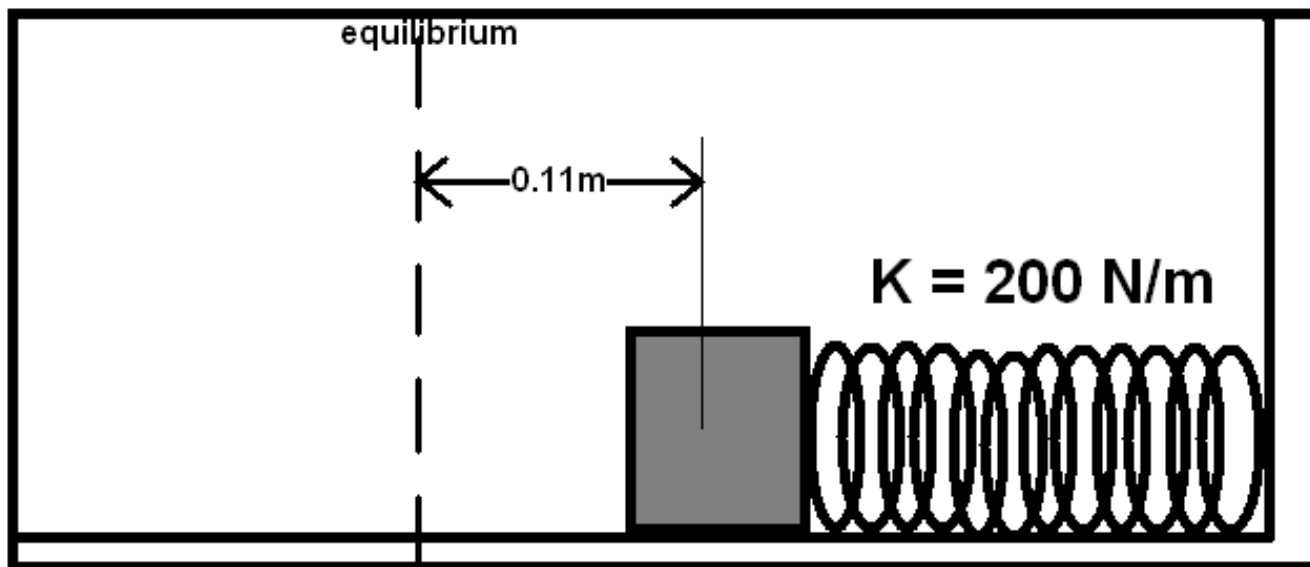
That curve forms the indicated triangle.

$$W = \frac{1}{2} (\text{base}) (\text{height})$$

$$W = \frac{1}{2} (x) (kx)$$

$$W = \frac{1}{2} kx^2$$

The energy imparted to the spring by this work must be stored in the Elastic Potential Energy (EPE) of the spring. Therefore, $EPE = \frac{1}{2}Kx^2$



Example 7: Determine the energy stored in a spring whose spring constant, k , is 200 N/m and which is compressed by a distance of 0.11m from its equilibrium length

$$\begin{aligned}
 \text{EPE} &= \frac{1}{2} kx^2 \\
 &= \frac{1}{2} (200 \text{ N/m})(.11\text{m})^2 \\
 &= (100 \text{ N/m})(.0121 \text{ m}^2) \\
 &= 1.21 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\
 &= 1.2 \text{ J}
 \end{aligned}$$

Example 8: A spring powered dart gun is used to shoot a dart straight upwards. The mass of the dart is 40 gm . The spring constant of the spring in the dart gun is 500 N/m and it is compressed a distance of 5.0 cm before being fired. Determine its velocity upon leaving the gun and the maximum height it attains (measured from the its initial location with the spring no longer compressed).

For the first part of the problem we need to determine the dart's velocity just at the moment it leaves the gun. At that point, all the EPE of the spring will have converted to KE and GPE.

$$E_0 + W = E_f$$

but $W = 0$ so

$$E_0 = E_f$$

the energy is either GPE or KE so

$$\text{EPE}_0 = \text{KE}_f + \text{GPE}_f$$

then substitute in the formulas for each

$$\frac{1}{2} kx^2 = \frac{1}{2} mv_f^2 + mgh_f$$

subtract mgh_f from both sides

$$\frac{1}{2} mv_f^2 = \frac{1}{2} kx^2 - mgh_f$$

divide both sides by m

$$\frac{1}{2} v_f^2 = \frac{1}{2} (k/m)x^2 - gh_f$$

double both sides

$$v_f^2 = (k/m)x^2 - 2gh_f$$

substitute in the given values

$$v_f^2 = (500\text{N/m}/.04\text{kg})(.05\text{m})^2 - 2(9.8 \text{ m/s}^2)(.05\text{m})$$

$$v_f^2 = 31 \text{ m}^2/\text{s}^2 - .98 \text{ m}^2/\text{s}^2$$

$$v_f = 5.5 \text{ m/s}$$

For the second part of the problem we need to use 5.5 m/s as the initial velocity and then determine from that how high it goes. We can use the work that we did in Example 6 to simplify the problem.

$$h_f = v_0^2 / g$$

$$h_f = (5.5 \text{ m/s})^2 / (9.8 \text{ m/s}^2)$$

$$h_f = (30.2 \text{ m}^2/\text{s}^2) / (9.8 \text{ m/s}^2)$$

$$h_f = 3.1 \text{ m}$$

Power

It is often important to know not only if there is enough energy available to perform a task but also how much time will be required. The concept of power allows us to answer these questions.

Power is defined as the rate that work is done. This is expressed mathematically in several forms. The most fundamental form is

Power = Work per unit time

$$P = W / t$$

Since $W = Fd_{\text{parallel}}$ this can also be expressed

$$P = (Fd_{\text{parallel}}) / t$$

Regrouping this becomes

$$P = F(d_{\text{parallel}} / t)$$

Since $v = d/t$

$$P = Fv_{\text{parallel}}$$

So power can be defined as the product of the force applied and the velocity of the object parallel to that force. A third useful expression for power can be derived from our original statement of the conservation of energy principle.

$$P = W / t$$

Since $W = E_f - E_0$

$$P = (E_f - E_0) / t$$

So the power absorbed by a system can be thought of as the rate at which the energy in the system is changing.

Units of Power

The SI unit for power can be derived from its basic equation.

$$P = W / t$$

The SI unit of energy is the Joule (J) and of time is the second (s). Therefore, the SI unit for power is a Joule per second (J / s). This has been designated the Watt (W) out of respect for James Watt (1736 - 1819), a pioneer in the development of the steam engine.

$$W = J/s$$

$$W = (N\cdot m)/s$$

$$W = ((kg\cdot m/s^2)(m))/s$$

$$W = J/s = kg\cdot m^2 / s^3$$

Example 9: What is the minimum power needed to lift a 120 kg mass 25 m straight up in 8.0 seconds?

$$P = W / t$$

but $W = Fd_{\text{parallel}}$ so

$$P = Fd_{\text{parallel}} / t$$

But the force needed to lift a mass is mg so

$$P = mgd_{\text{parallel}} / t$$

substitute in the given values

$$P = (120\text{kg})(9.8 \text{ m/s}^2)(25\text{m})/(8.0\text{s})$$

$$P = 3675 \text{ kg}\cdot\text{m}^2/\text{s}^3$$

$$P = 3700 \text{ W}$$

Example 10: If a motor can supply 45000 W of power, with what velocity can it lift a 1200 kg elevator car filled with up to 8 110 kg people.

First we'll need to calculate the total mass of the car loaded with the maximum number of people.

$$m_{\text{total}} = m_{\text{car}} + 8 (m_{\text{person}})$$

$$m_{\text{total}} = 1200\text{kg} + 8 (110\text{kg})$$

$$m_{\text{total}} = 1200\text{kg} + 880\text{kg}$$

$$m_{\text{total}} = 2080 \text{ kg}$$

Then

$$P = Fv_{\text{parallel}}$$

Recognizing that $F = mg$ and $v = v_{\text{parallel}}$

$$P = (mg)v$$

Dividing both sides by mg yields

$$v = P/(mg)$$

Substituting values yields

$$v = 45000\text{W}/((2080\text{kg})(9.8 \text{ m/s}^2))$$

$$v = 2.21 \text{ (kg}\cdot\text{m}^2/\text{s}^3)/(\text{kg}\cdot\text{m/s}^2)$$

$$v = 2.2 \text{ m/s}$$

Simple Machines

Simple machines make it possible for us to accomplish what might otherwise be difficult or impossible. They have been used throughout history. Initially they were hand powered. As technology evolved it proved possible to use other sources of power such as water, wind, steam, internal combustion engines and electric motors to drive these machines. However, regardless of their power source, simple machines still serve the same function of allowing the best match between the source of energy and the work that needs to be done.

Simple machines include the inclined plane, screw, lever and pulley. All of them work on the basis of the equation that

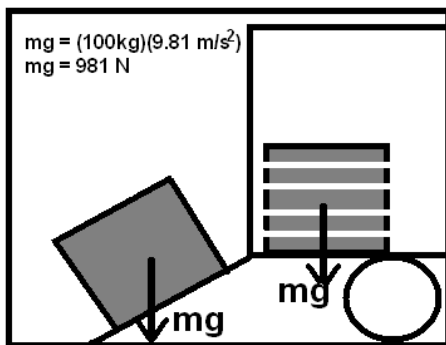
$$W = Fd_{\text{parallel}}$$

In all these cases the force will be made parallel to the distance moved so we can simplify this expression to

$$W = Fd$$

Machines cannot create energy. Rather, they allow us to use energy more effectively by varying the force and distance involved while leaving the product of those two factors, the work, unchanged.

For instance, let's say that you needed to lift a 100 kg (220 lb) piece of equipment from the ground to the back of a truck, 1.0 m above the ground. That would be virtually impossible for most people without the aid of a simple machine. An average person could not lift 220 lbs an inch, let alone a meter. The required force, in SI units, would be given by



$$E_0 + W = E_f$$

$$W = E_f$$

$$Fd = E_f$$

$$Fd = mgh_f$$

$$F = mgh_f / d$$

$$F = (100\text{kg})(9.8 \text{ m/s}^2)/1.0\text{m}$$

$$F = 980 \text{ N}$$

We can take $E_0 = 0$ so

and $W = Fd$ to get

But $E_f = \text{GPE} = mgh_f$ so

Solve for F by dividing both sides by d

Substituting values

However, if a 5.0 m long inclined plane (using low friction bearings) is extended from the back of the truck to the ground, the piece of equipment could be pushed up the ramp. The effect of doing this is to increase d from 1.0 m to 5.0 m. Let's calculate the force needed to push the equipment up the ramp (assuming no friction).

$$E_0 + W = E_f$$

We can take $E_0 = 0$ so

$$W = E_f$$

and $W = Fd$ to get

$$Fd = E_f$$

But $E_f = \text{GPE} = mgh_f$ so

$$Fd = mgh_f$$

Solve for F by dividing both sides by d

$$F = mgh_f / d$$

Substitute in values

$$F = (100\text{kg})(9.8 \text{ m/s}^2)/5.0\text{m}$$

$$F = 200 \text{ N}$$

Since d is now 5 times larger and W stays the same (this is where the frictionless part of the plane is important) the needed force becomes 1/5 as large. Instead of requiring 980N, the task can be accomplished with a force of 200N (44 lb). Even with the addition of friction, the task would go from being impossible to relatively easy. The tradeoff of having to push the box five times as far is well worth it if the box is sufficiently heavy.

This is the same principle that underlies all simple machines. When the goal is to reduce the amount of force needed, W is kept constant while d is increased. The result is that the force is decreased by the same multiple that the distance is increased.

In some cases the goal is to increase the force. For instance, a lever is used to move a heavy boulder. A lever consists of a rigid rod or board atop a fulcrum. By making the distance from your hands to the fulcrum larger than the distance from the fulcrum to the object, the force exerted on that object is magnified.

Simple machines can work in either direction. If you push on the longer side of the lever you magnify the force and reduce the distance. However, you could also push on the shorter side to reduce the force and increase the distance. In either case, in a perfectly efficient simple machine, the product of the force and distance is the same. Therefore, the best way to look at these problems is to use the equation for work on both sides and recognize that the conservation of energy principle assures that they are equal. The work that I put in on my side of the fulcrum results in work being done to the object on the other side.

$$W_{\text{out}} = W_{\text{in}}$$

Substitute in $W = Fd$ (F and d are parallel) on both sides of the equation

$$F_{\text{out}}d_{\text{out}} = F_{\text{in}}d_{\text{in}}$$

Divide both sides by d_{out}

$$F_{\text{out}} = F_{\text{in}}d_{\text{in}} / d_{\text{out}}$$

Regroup the terms

$$F_{\text{out}} = F_{\text{in}} (d_{\text{in}} / d_{\text{out}})$$

Since d_{in} is proportional to the length of the lever on my side of the fulcrum and d_{out} is proportional to the length of the lever on the other side of the fulcrum, this machine magnifies the force by a factor equal to the ratio of those two lengths. The tradeoff is that although I've increased the amount of force, I have decreased the distance I will move the object by the same factor.

Example 11: I need to lift a 500 kg (1100 lb boulder) a few cm off the ground. The closest that I can get the fulcrum to the bottom of the boulder is 10 cm. How long will the lever need to be so that I could move the boulder by pushing down with all my weight, 500N?

$$W_{\text{out}} = W_{\text{in}}$$

Substitute in $W = Fd$ (F and d are parallel) on both sides of the equation

$$F_{\text{out}}d_{\text{out}} = F_{\text{in}}d_{\text{in}}$$

Divide both sides by F_{in}

$$F_{\text{out}}d_{\text{out}} / F_{\text{in}} = d_{\text{in}}$$

Or

$$d_{\text{in}} = F_{\text{out}}d_{\text{out}} / F_{\text{in}}$$

Substituting in the values with $F_{\text{out}} = m_{\text{boulder}}g$

$$d_{\text{in}} = (500\text{kg})(9.8 \text{ m/s}^2)(.1 \text{ m})/500\text{N}$$

$$d_{\text{in}} = 0.98 \text{ m}$$

But the lever consists of both sides, $d_{\text{in}} + d_{\text{out}}$

$$L = d_{\text{in}} + d_{\text{out}}$$

$$L = .1\text{m} + .98\text{m}$$

$$L = 1.1\text{m}$$

Chapter Questions

1. A block is suspended from a string; does the gravitational force do any work on it?
2. What is the difference between work done by the gravitational force on descending and ascending objects?
3. A woman climbs up stairs; does she do any work? Does she do any work standing in an ascending elevator?
4. What happens to an object's velocity if there is work done by a friction force? Why?
5. An object is suspended from a spring and is at equilibrium; does the elastic force do any work?
6. It is known that water applies some pressure on a container; does water do any work in this case?
7. What kind of energy does a flying bullet have?
8. A stone is thrown vertically up. What kind of energy did the stone have initially? What happens to this energy as the stone ascends?
9. A steel ball and an aluminum ball of equal volume are located at the same altitude. Which ball has greater gravitational potential energy?
10. What happens to the gravitational potential energy of an object when it moves up? When it moves down?
11. Is it possible for a static friction force to do mechanical work? Give an example?
12. Can kinetic energy ever be negative? Explain.
13. Describe the energy transformation that takes place when a small mass oscillates at the end of a light string.
14. Describe the energy transformations that take place when a small mass oscillates at the end of an elastic spring.
15. An elevator is lifted vertically upwards at a constant speed. Is the net work done on the elevator negative, positive, or zero? Explain.
16. Can the net work done on an object during a displacement be negative? Explain.

Chapter Problems

Work:

Examples

1. A car engine applies a force of 65 kN, how much work is done by the engine as it pushed a car a distance of 75 m?
2. A force does 30000 J of work along a distance of 9.5 m. Find the applied force.
3. How high can a 40 N force move a load, when 395 J of work is done?
4. How much work is required to lift a 500 kg block 12 m?

Class Work

5. A 60 N force is applied over distance of 15 m. How much work was done?
6. A railroad car is pulled through the distance of 960 m by a train that did 578 kJ of work during this pull. How much force did the train supply?
7. A boy pulls a sled at a constant speed 0.6 m/s by applying a force of 350 N. How much work will be done during 1800 s?

Homework

8. A light plane travels a distance of 150 m along a runway before takeoff. Find the work done by the plane engine if it is applying a force of 13500 N.
9. A horse pulls a carriage by applying 450 N of force. Find the traveled distance if the horse did 89 kJ of work.
10. A truck travels at a constant speed of 45 m/s. How much work did the truck engine do during a 2 hour period if it supplied a force of 25 kN of force.
11. Airflow lifts a 3.6 kg bird 50 m up. How much work was done by the flow?

Gravitational Potential Energy (GPE):

Examples

12. A 2.4 kg toy falls from 2 m to 1 m. What is the change in GPE?
13. If (on earth) an object falls 18 m and loses 36 J of GPE. What is the object's mass?
14. A 1 Kg object loses 20 J of GPE as it falls. How far does it fall?
15. A small, 3 kg weight is moved from 5 m from the ground to 8 m. What is the change in potential energy?

Class Work

16. An 80 kg person falls 60 m off of a waterfall. What is her change in GPE?

17. A Gravitational Potential Energy (GPE) Sensor attached to a 12 Kg ball changes from 12 J to 22 J, by height change alone. What is the change in height?
18. A man rides up in an elevator 12 m. He gains 6500 J of gravitational potential energy. What is the man's mass?
19. When a 5 kg rock is dropped from a height of 6 m on Planet X, it loses 24 J of GPE. What is the acceleration due to gravity on Planet X?

Homework

20. What is the gravitational potential energy of a 450 Kg car at the top of a 25 m parking garage?
21. What is the change in gravitational potential energy of a 45 kg weight that is moved from 2 m to 18 m on earth? What is it on the moon ($g = 1.6 \text{ m/s}^2$)?
22. A 0.25 kg book falls off a 2 m shelf on to a 0.5 m chair. What was the change in GPE?
23. A 60 kg girl falls off of a waterfall and loses 10 kJ of GPE. What was her height?
24. When a 0.5 kg rock is dropped from a height of 12 m on Planet Z, it loses 45 J of GPE. What is the acceleration due to gravity on Planet Z?

Kinetic Energy (KE)

Examples

25. How much kinetic energy does an 80 kg man have while running at 1.5 m/s?
26. A bird flies at a speed of 2.3 m/s if it has 14 J of kinetic energy, what is its mass?
27. A child does 12 J of work pushing his 3 kg toy truck. With what velocity does the toy move after the child is done pushing?

Class Work

28. A 6 kg object has a speed of 24 m/s. What is its kinetic energy?
29. A rock hits the ground with a speed of 7 m/s and a kinetic energy of 100 J. What is the rock's mass?
30. A bullet is fired into a 12 kg block of wood. After the bullet stops in the block of wood the block has 29 J of kinetic energy. At what speed is the block moving?

Homework

31. How much kinetic energy does a 4 Kg cat have while running at 9 m/s?

32. What is the mass of an object moving with a speed of 4 m/s and a kinetic energy of 2000 J?
33. A 400 Kg car has 1.8×10^5 J of kinetic energy. How fast is it moving?
34. How fast is a 3 Kg toy car with 20 J of kinetic energy moving?
35. A student runs to physics class with a speed of 6 m/s. If the student has 880 J of kinetic energy, what is her mass?
36. What is the speed of a 1200 kg car moving with a kinetic energy of 15 kJ?

Elastic Potential Energy

Examples

37. A spring with a spring constant of 120 N/m stretches by 0.02 m. What is the potential energy of the spring?
38. An elastic spring stores 45 J of potential energy when it is stretched by 2 cm. What is the spring constant?
39. A 50 N force causes a spring to compress 0.09 m. What is the spring constant? What is the potential energy of the spring?

Class Work

40. An 80 N force causes a spring to compress 0.15 m. What is the spring constant? What is the potential energy of the spring?
41. A spring with a spring constant of 200 N/m stretches by 0.03 m. What is the potential energy of the spring?
42. A spring stores 68 J of potential energy when it is stretched by 6 cm. What is the spring constant?
43. A spring with spring constant 60 N/m has 24 J of EPE stored in it. How much is it compressed?

Homework

44. How much energy is stored in a spring with a spring constant of 150 N/m when it is compressed 2 cm?
45. A spring with spring constant 175 N/m has 20 J of EPE stored in it. How much is it compressed?
46. A spring stores 96 J of potential energy when it is stretched by 5 cm. What is the spring constant?
47. A 0.20 kg mass attached to the end of a spring causes it to stretch 3.0 cm. What is the spring constant? What is the potential energy of the spring?

48. A 5 kg mass, hung onto a spring, causes the spring to stretch 7.0 cm. What is the spring constant? What is the potential energy of the spring?

Mixed Problems

Class Work

49. A 5 kg rock is raised 28 m above the ground level. What is the change in its potential energy?
50. A 65 kg cart travels at constant speed of 4.6 m/s. What is its kinetic energy?
51. What is the potential energy of stretched spring, if the spring constant is 40 N/m and the elongation is 5 cm?
52. A 3.5 kg object gains 76 J of potential energy as it is lifted vertically. Find the new height of the object?
53. A spring has a spring constant of 450 N/m. How much must this spring be stretched to store 49 J of potential energy?
54. A 60 kg runner has 1500 J of kinetic energy. How fast is he moving?

Homework

55. A spring with spring constant 270 N/m has 5 J of energy stored in it. How much is it compressed?
56. A 0.02 kg rock strikes the ground with 0.36 J of kinetic energy. What was its velocity?
57. A woman riding a bicycle has a kinetic energy of 3600 J when traveling at a speed of 12 m/s. What is her mass?
58. On Planet X a 0.5 kg space rock falls a distance of 2.5 meters and loses 20 J of energy. What is the gravity on Planet X?
59. A 50 kg skydiver loses 2400 kJ of energy during a jump. What was her change in height?
60. A child compresses his spring gun 1 cm. If 3 mJ of energy are stored in the spring, what is the spring constant?
61. A stone is thrown vertically up with a speed of 14 m/s, and at that moment it had 37 kJ of kinetic energy. What was the mass of the stone?

Conservation of Energy

Examples

62. A spring gun with a spring constant of 250 N/m is compressed 5 cm. How fast will a 0.025 kg dart go when it leaves the gun?

63. A student uses a spring (with a spring constant of 180 N/m) to launch a marble vertically into the air. The mass of the marble is 0.004 kg and the spring is compressed 0.03 m. How high will the marble go?
64. A student uses a spring gun (with a spring constant of 120 N/m) to launch a marble vertically into the air. The mass of the marble is 0.002 kg and the spring is compressed 0.04 m.
- How high will the marble go?
 - How fast will it be going when it leaves the gun?
65. A roller coaster has a velocity of 25 m/s at the bottom of the first hill. How high was the hill?

Class Work

66. How much work is needed to lift a 3 kg mass a distance of 0.75 m?
67. An arrow is fired vertically upwards by a bow and reaches an altitude of 134 m. Find the initial speed of the arrow on the ground level.
68. A student uses a spring to launch a marble vertically in the air. The mass of the marble is 0.002 kg and when the spring is stretched 0.05 m it exerts a force of 10 N. What is the maximum height the marble can reach?
69. A children's roller coaster is released from the top of a track. If its maximum speed at ground level is 8 m/s, find the height it was released from.
70. A student uses a spring with a spring constant of 130 N/m in his projectile apparatus. When 56 J of potential energy is required to launch the projectile to a certain height, what is the compression in the spring?
71. How much work must be done to accelerate an 8×10^5 kg train: a) from 10 m/s to 15 m/s; b) from 15 m/s to 20 m/s; c) to a stop an initial speed of 20 m/s?

Homework

72. How much work is done in accelerating a 2000 kg car from rest to a speed of 30 m/s?
73. A rock is dropped from a height of 2.7 m. How fast is it going when it hits the ground?
74. A roller coaster is released from the top of a track that is 125 m high. Find the roller coaster speed when it reaches ground level.
75. A 1500 kg car, moving at a speed of 20 m/s comes to a halt. How much work was done by the brakes?
76. A projectile is fired vertically upward with an initial velocity of 190 m/s. Find the maximum height of the projectile.
77. A spring gun is used to project a 0.5 kg ball, in order to perform this experiment the spring was initially compressed by 0.005 m. Find the ball's speed when it leaves the gun, if the spring constant is 395 N/m.

78. A student uses a spring loaded launcher to launch a marble vertically in the air. The mass of the marble is 0.003 kg and the spring constant is 220 N/m. What is the maximum height the marble can reach (a) when compressed 2 cm? (b) when compressed 4 cm?

Power

Examples

79. A heat engine does 23 kJ of work during 1800 s. Find the power supplied by the engine.
80. How much work is done by 15 kW engine during 3.5 h?
81. How long must a 400 W electrical engine work in order to produce 300 kJ of work?
82. How much power is required when using a 12 N force to push an object at a velocity of 3 m/s?

Class Work

83. An elevator motor in a high-rise building can do 3500 kJ of work in 5 min. Find the power developed by the motor.
84. It takes 500 W of power to move an object 96 m in 12 s. What force is being applied to the object?
85. A heat turbine can generate a maximum power of 250 MW. How much work can the turbine do in 7.8 h?
86. How much time is required for a car engine to do 278 kJ of work, if its maximum power is 95 kW?

Homework

87. How much time is required for a elevator to lift a 2000 kg load up 28 m from the ground level, if the motor can produce 13 kW of power?
88. A 50 kW pump is used to pump up water from a mine that is 50 m deep. Find the mass of water that can be lifted by the pump in 1.4 h.
89. Some scientists calculated that a whale can develop 150 kW of power when it is swimming under the water surface at a constant speed 28 km/h. Find the resistance force of the water exerted on the whale.
90. A tractor travels at constant speed of 21.6 km/h. Find the power supplied by the engine if it can supply a maximum force of 467 kN.
91. A 7.35 kW lathe can move an iron block at a constant speed by applying a force of 5.56 kN. Find the speed of the block.

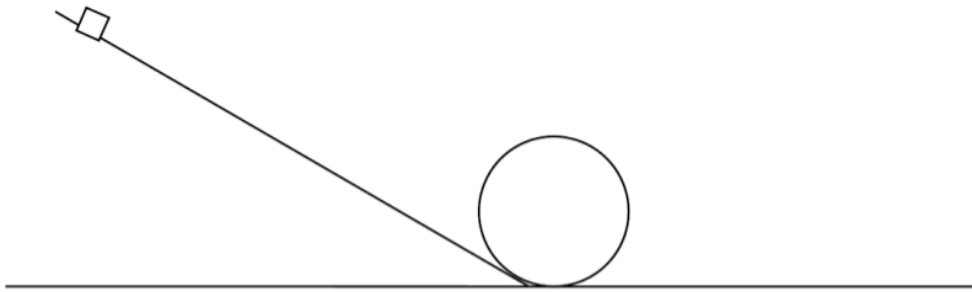
General Problems

92. A 255 N force is applied to a 46 kg box that is located on a flat horizontal surface. The coefficient of kinetic friction between the box and the surface is 0.3.

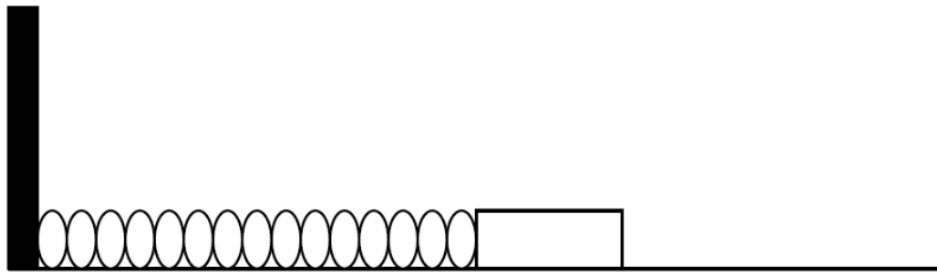
- a. Sketch a free-body diagram and show all the applied forces.
- b. Find the acceleration of the box
- c. How far the box will go in 10 s?
- d. What will be the velocity at the end of this distance?
- e. Find the kinetic energy after 10 s of traveling.
- f. How much work is done during the first ten seconds by each of the following; the applied force, friction force, normal force, gravitational force and net force?
- g. Compare the work done by the net force and the final kinetic energy.

93. A worker pushes a 50 kg crate a distance of 7.5 m across a level floor. He pushes it at a constant speed by applying a constant horizontal force. The coefficient of kinetic friction between the crate and the floor is 0.15.

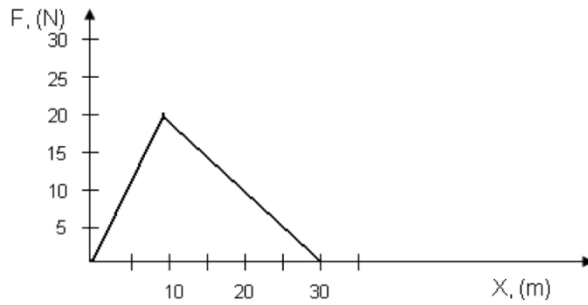
- a. Find the magnitude of the applied force.
- b. How much work did the worker do on the crate?
- c. How much work did the friction force do on the crate?
- d. How much work did the normal force do on the crate?
- e. How much work did the gravitational force do on the crate?
- f. What was the total work done on the crate?
- g. What was the change in the kinetic energy of the crate?



94. A small block, with a mass of 250 g, starts from rest at the top of the apparatus shown above. It then slides without friction down the incline, around the loop and then onto the final level section on the right. The maximum height of the incline is 80 cm, and the radius of the loop is 15 cm.
- Find the initial potential energy of the block
 - Find the velocity the block at the bottom of the loop
 - Find the velocity of the block at the top of the loop.
 - What is the normal force on the block at the lowest point of the loop?
 - What is the normal force on the block at the highest point of the loop?

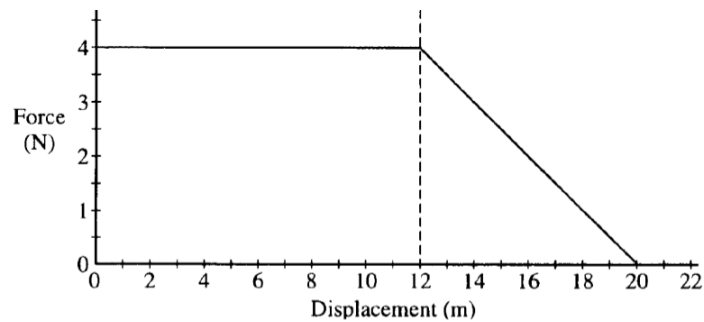


95. A 0.8 kg block is attached to the end of a spring whose spring constant is 85 N/m. The block is placed on a frictionless tabletop, given an initial displacement of 3.5 cm and then released.
- What type of energy did the block-spring system initially have?
 - Find the magnitude of this energy.
 - How does the total energy of the block-spring system change as the block is pushed across the frictionless surface? Explain.
 - Find the maximum velocity of the block.



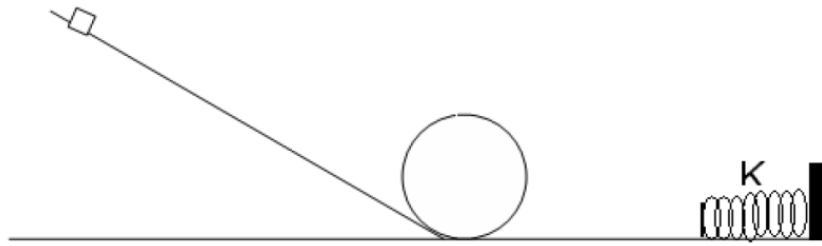
96. An external horizontal force, F , is applied to a 2.5 kg toy car as it moves in a straight line. The force varies with the car's displacement as shown above. Using the graph answer the following questions.

- How much work is done by the applied force while the car move the first 10 m?
- Determine the kinetic energy of the car when it passes the 10 m point?
- What is the velocity of the car when it passes the 10m point?
- What is the total work done by the force in the process of displacing the car the first 30 m?
- What is the kinetic energy of the car when it is 30 m from the origin?
- What is the velocity of the car when it is 30 m from the origin?

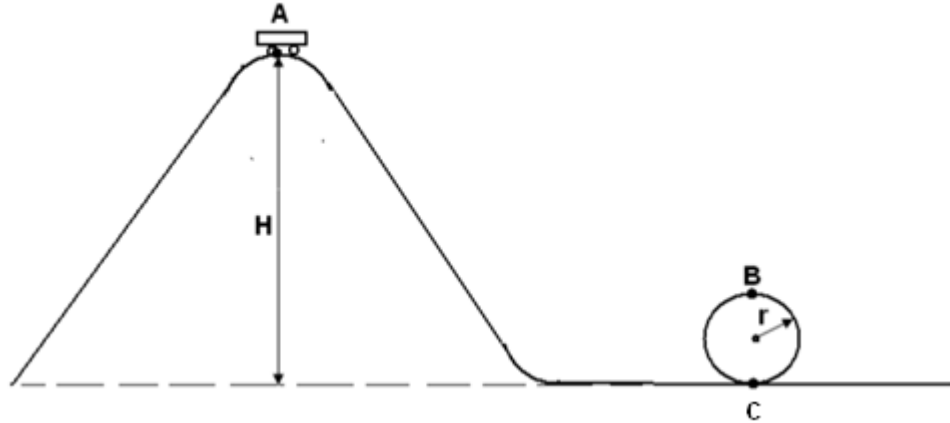


97. A 2 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the diagram above. The object starts from rest at displacement $x = 0$ and time $t = 0$ and is travels a distance 20 m. Find the following.

- The acceleration of the object when it has traveled 5 m.
- The time taken for the object to be move the first 12 m.
- The amount of work done by the net force in displacing the object the first 12 m.
- The speed of the object at a displacement of 12 m.
- The speed of the object at a displacement 20 m.



98. A small block, with a mass of 1.5 kg, starts from rest at the top of the apparatus shown above. It then slides without friction down the incline, around the loop and then onto the final level section on the right. It then collides with a spring which momentarily brings the block to a stop. The maximum height of the incline is 2.5 m, the radius of the loop is 0.9 m and the spring constant is 90 N/m.
- Find the initial potential energy of the block.
 - Find the velocity of the block at the top of the loop.
 - Find the velocity of the block after it goes around the loop, on the flat section of the path.
 - How much will the block compress the spring before momentarily coming to a stop?



99. A roller coaster of mass 500 kg starts its ride from rest at point A. Point A is located at a height of 70 m above the lowest point on the track. The car rolls down the incline and follows the track around a loop of radius 15 m. Ignore friction force.

- How much work is required to bring the car to point A?
- Calculate the speed of the car at point C.
- On the figure of the car below, draw and label vectors to represent the forces acting on the car it is at point C.



- Calculate the speed of the car at point B.
- On the figure of the car below, draw and label vectors to represent the forces acting on the car it is upside down at point B.



- Now suppose that friction is not negligible. How would friction affect the answers in (a) and (c)?

1) 4875000 J	37) 0.024 J	62) 5 m/s	93) a) 73.5 N
2) 3158 N	38) 225000 N/m	63) 2.07 m	b) 551.25 J
3) 9875 m	39) 555.6 N/m	64) a) 4.9 m	c) 551.25 J
4) 58800 J	2.25 J	b) 9.8 m/s	d) 0 J
5) 900 J	40) 533.3 N/m	65) 31.9m	e) 0 J
6) 602.08 N	6 J	66) 22.05 J	f) 0 J
7) 378000 J	41) 0.09 J	67) 51.2 m/s	g) 0 J
8) 2025000 J	42) 37778 N/m	68) 12.8 m	94) a) 1.96 J
9) 198 m	43) 0.89 m	69) 3.27 m	b) 3.96 m/s
10) 8×10^9 J	44) 0.03 J	70) 0.93 m	c) 3.13 m/s
11) 1764 J	45) 0.48 m	71) a) 5.3125×10^7 J	d) 28.586 N
	46) 76800 N/m	b) 7.4375×10^7 J	e) 13.883 N
12) 23.52 J	47) 65.3 N/m	c) 1.7×10^8 J	95) a) EPE
13) 0.20 kg	0.0294 J	72) 900,000 J	b) 0.052 J
14) 2.04 m	48) 700 N/m	73) 7.27 m/s	c) EPE ↔ KE
15) 88.2 J	1.715 J	74) 49.5 m/s	d) 0.36 m/s
16) 47040 J		75) 300,000 J	96) a) 100 J
17) 0.085 m	49) 1372 J	76) 1842 m	b) 100 J
18) 55.3 kg	50) 687.7 J	77) 0.14 m/s	c) 8.9 m/s
19) 0.8 m/s^2	51) 0.05 J	78) a) 1.5 m	d) 300 J
20) 110250 J	52) 2.22 m	b) 6.0 m	e) 300 J
21) $7056 J_{\text{Earth}}$	53) 0.45 m	79) 12.8 W	f) 15.5 m/s
$1152 J_{\text{Moon}}$	54) 7.07 m/s	80) 3.15×10^6 J	97) a) 2 m/s^2
22) 3.675 J	55) 0.19 m	81) 750s	b) 3.46 s
23) 17 m	56) 6 m/s	82) 36 W	c) 48 J
24) 7.5 m/s^2	57) 50 kg	83) 11667 W	d) 6.93 m/s
	58) 16 m/s^2	84) 62.5N	e) 8 m/s
25) 90 J	59) 4900 m	85) 1.17×10^{11} J	98) a) 36.75 J
26) 5.29 kg	60) 60 N/m	86) 2.9 s	b) 3.7 m/s
27) 2.83 m/s	61) 0.38 kg	87) 42.2 s	c) 7 m/s
28) 1728 J		88) 8571.42 kg	d) 0.9 m
29) 4.08 kg		89) 5.357 N	99) a) 343000 J
30) 2.20 m/s		90) 1.008×10^{10} W	b) 37.04 m/s
31) 162 J		91) 1.32 m/s	c) F_N up, mg down
32) 250 kg		92) a) F_N , mg, f, F_A	d) 28 m/s
33) 30.4 m/s		b) 2.6 m/s^2	e) F_N down, mg down
34) 3.65 m/s		c) 130 m	f) less velocity at B and C
35) 48.9 kg		d) 26 m/s	
36) 5 m/s		e) 15548 J	
		f) 33150 J	
		17581 J	
		0 J	
		0 J	
		15569 J	
		g) they are the same	

Chapter Problems Work Examples

1. $F=65\text{KN}$ $W=FD$
 $D=75\text{m}$ $(65 \times 10^3\text{N})(75\text{m})$
 $W=?$ **$W=4875000$**

2. $W=30000\text{J}$ $W=FD$
 $D=9.5\text{m}$ $F=W/D$
 $F=?$ $(30000\text{J}/9.5\text{m})$
 $F=3157.9\text{N}$

3. $F=40\text{N}$ $W=FD$
 $W=395\text{J}$ $D=W/F$
 $D=?$ $(395\text{J})/(40\text{N})$
 $D=9.875\text{m}$

4. $W=?$ $F=Mg$ $W=FD$
 $M=500\text{kg}$ $(9.8)(500\text{kg})$ $(4900\text{N})(12\text{m})$
 $D=12\text{m}$ $F=4900\text{N}$ **$W=58800$**

Class Work

5. $F=60\text{N}$ $W=FD$
 $D=15\text{m}$ $(60\text{N})(15\text{m})$
 $W=?$ **$W=900\text{N}$**

6. $D=960\text{m}$ $W=FD$
 $W=578\text{kJ}$ $F=W/D$
 $F=?$ $(578\text{kJ})/(960\text{m})$
 $F=602.08\text{N}$

7. $V=.6\text{m/s}$ $x=vt$ $W=FD$
 $F=350\text{N}$ $(.6\text{m/s})(1800\text{s})$ $(350\text{N})(1080\text{m})$
 $T=1800\text{s}$ $x=1080\text{m}$ **$W=378\text{kJ}$**
 $W=?$

Homework

8. $D=150\text{m}$ $W=FD$
 $F=13,500\text{N}$ $(150\text{m})(13500\text{N})$
 $W=?$ **$W=2025000\text{J}$**

9. $F=450\text{N}$ $W=FD$
 $D=?$ $D=W/F$
 $W=89\text{kJ}$ $(89\text{kJ})/(450\text{N})$
 $D=197.78\text{m}$

Homework Cont'd

10. $V=45\text{m/s}$ $D=vt$ $W=FD$
 $W=?$ $(45\text{m/s})(7200\text{s})$ $(25\text{kN})(324000)$
 $t=2\text{hr}=7200\text{s}$ $D=324,000\text{m}$ **$W=8.1 \times 10^9\text{J}$**
 $F=25\text{kN}$

100. $m=3.6\text{kg}$ $F=mg$ $W=FD$
 $D=50\text{m}$ $(3.6\text{kg})(9.8)$ $(35.28\text{N})(50\text{m})$
 $W=?$ $F=35.28\text{N}$ **$W=1764\text{J}$**

Gravitational Potential Energy

h. $m=2.4\text{kg}$ $GPE=Mgh$ $GPE=Mgh$
 $x_i=2\text{m}$ $(2.4\text{kg})(9.8)(1\text{m})$ $(2.4\text{kg})(9.8)(2\text{m})$
 $x_f=1\text{m}$ $GPE=23.52\text{J}$ $GPE=47.04\text{J}$
 $GPE=?$ $47.04-23.52=\mathbf{23.52\text{J}}$

13. $h=18\text{m}$ $GPE=mgh$
 $w=36\text{J}$ $36\text{J}=M(9.8)(18\text{m})$
 $M=?$ $M=36\text{J}/(9.8)(18)$
 $M=.204\text{kg}$

14. $h=?$ $GPE=mgh$
 $GPE=20\text{J}$ $20\text{J}=(1\text{kg})(9.8)h$
 $M=1\text{kg}$ $H=20\text{J}/(9.8)$
 $H=2.04\text{m}$

15. $m=3\text{kg}$ $GPE=Mgh$ $GPE=Mgh$
 $x_i=5\text{m}$ $(2.4\text{kg})(9.8)(5\text{m})$ $(2.4\text{kg})(9.8)(8\text{m})$
 $x_f=8\text{m}$ $GPE=147\text{J}$ $GPE=235.2\text{J}$
 $GPE=?$ $235.2-147=\mathbf{88.2\text{J}}$

Classwork

16. $m=80\text{kg}$ $GPE=Mgh$
 $D=60\text{m}$ $(80\text{kg})(9.8)(60)$
 $GPE=?$ **$GPE=47,040\text{J}$**

17. $h=?$ $GPE=mgh$
 $GPE=10\text{J}$ $10\text{J}=(12\text{kg})(9.8)h$
 $M=12\text{kg}$ $H=10\text{J}/(9.8)(12\text{kg})$
 $H=.085\text{m}$

18. $h=12\text{m}$ $GPE=mgh$
 $GPE=6500\text{J}$ $6500=M(9.8)(12\text{m})$
 $M=?$ $M=6500\text{J}/(9.8)(12)$

Elastic Potential Energy Problems

37. $K=120\text{N/M}$ $EPE=1/2Kx^2$
 $x=.02\text{m}$ $1/2(120\text{N/M})(.02)^2$
 $EPE=?$ **EPE=.024J**

38. $EPE=45\text{J}$ $EPE=1/2Kx^2$
 $x=2\text{cm}=.02\text{m}$ $45\text{J}=1/2k(.02)^2$
 $v=?$ $k=90/.0004$
K=225000N/M

39. $F=50\text{N}$ $F=Kx$ $EPE=1/2Kx^2$
 $x=.09$ $K=F/x$ $1/2(555.56)(.09)^2$
 $50/.09$ **EPE=2.25J**
K=555.56N/M

Class Work

40. $F=80\text{N}$ $F=Kx$ $EPE=1/2Kx^2$
 $x=.15$ $K=F/x$ $1/2(533.33)(.15)^2$
 $80/.15$ **EPE=5.99J**
K=533.33N/M

41. $K=200\text{N/M}$ $EPE=1/2Kx^2$
 $x=.03\text{m}$ $1/2(200\text{N/M})(.03)^2$
 $EPE=?$ **EPE=.09J**

42. $EPE=68\text{J}$ $EPE=1/2Kx^2$
 $x=6\text{cm}=.06\text{m}$ $68=1/2k(.06)^2$
 $k=?$ $K=136/.0036$
K=3,777.78N/M

43. $K=60\text{N/M}$ $EPE=1/2Kx^2$
 $EPE=24\text{J}$ $24\text{J}=1/2(60)x^2$
 $x=?$ $x=\sqrt{(24)/(30)}$
x=.894m

Homework

44. $K=150\text{N/M}$ $EPE=1/2Kx^2$
 $x=.02\text{m}$ $1/2(150\text{N/M})(.02)^2$
 $EPE=?$ **EPE=.03J**

45. $K=175\text{N/M}$ $EPE=1/2Kx^2$
 $EPE=20\text{J}$ $20\text{J}=1/2(175)x^2$
 $x=?$ $x=\sqrt{(40)/(175)}$
x=.478m

Homework Cont'd

46. $EPE=96\text{J}$ $EPE=1/2Kx^2$
 $x=5\text{cm}=.05\text{m}$ $96=1/2k(.05)^2$
 $k=?$ $K=192/.0025$
K=76,800N/M

47. $F=?$ $F=mg$ $F=Kx$
 $x=.03\text{m}$ $(.2\text{kg})(9.8)$ $K=F/x$
 $m=.2\text{kg}$ $F=1.96\text{N}$ $1.96\text{N}/.03,$
K=65.33N/M

$EPE=1/2Kx^2$
 $1/2(65.33\text{N/M})(.03)^2$
EPE=.0294J

48. $F=?$ $F=mg$ $F=Kx$
 $x=.07$ $(5\text{kg})(9.8)$ $K=F/x$
 $m=5\text{kg}$ $F=49\text{N}$ $49\text{N}/.07,$
K=700N/M

$EPE=1/2Kx^2$
 $1/2(700\text{N/M})(.07)^2$
EPE=1.715J

Mixed Problems

Class Work

49. $m=5\text{kg}$ $GPE=Mgh$
 $h=28$ $(5\text{kg})(9.8)(28\text{m})$
 $GPE=?$
GPE=1372J

50. $M=65\text{kg}$ $KE=1/2mv^2$
 $v=4.6\text{m/s}$ $1/2(65\text{kg})(4.6\text{m/s})^2$
KE= 687.7J

51. $K=40\text{N/M}$ $EPE=1/2Kx^2$
 $x=.05\text{m}$ $1/2(40\text{N/M})(.05)^2$
 $EPE=?$ **EPE=.05J**

52. $h=?$ $GPE=mgh$
 $GPE=76\text{J}$ $76\text{J}=(3.5\text{kg})(9.8)h$
 $M=3.5\text{kg}$ $H=76\text{J}/(9.8)(3.5)$
H=2.22m

Class Work Cont'd

53. $K=450\text{N/M}$ $EPE=1/2Kx^2$

$$\begin{aligned} \text{EPE} &= 46\text{J} & 46\text{J} &= \frac{1}{2}(450)x^2 \\ x &=? & x &= \sqrt{(2)(46)/(450)} \\ & & \mathbf{x} &= \mathbf{.45\text{m}} \end{aligned}$$

$$\begin{aligned} \text{KE} &= 37\text{J} & 37\text{J} &= \frac{1}{2}m(14\text{m/s})^2 \\ M &=? & m &= (2)37\text{J}/(14\text{m/s}^2) \\ & & \mathbf{M} &= \mathbf{.377\text{kg}} \end{aligned}$$

$$\begin{aligned} 54. \text{KE} &= 1500\text{J} & \text{KE} &= \frac{1}{2}mv^2 \\ m &= 60\text{kg} & 1500\text{J} &= \frac{1}{2}(60\text{kg})v^2 \\ v &=? & v &= \sqrt{(2)(1500)/(60\text{kg})} \\ & & \mathbf{v} &= \mathbf{7.07\text{m/s}} \end{aligned}$$

Homework

$$\begin{aligned} 55. K &= 270\text{N/M} & \text{EPE} &= \frac{1}{2}Kx^2 \\ \text{EPE} &= 5\text{J} & 5\text{J} &= \frac{1}{2}(270)x^2 \\ x &=? & x &= \sqrt{(2)(5)/(270)} \\ & & \mathbf{x} &= \mathbf{.19\text{m}} \end{aligned}$$

$$\begin{aligned} 56. \text{KE} &= .36\text{J} & \text{KE} &= \frac{1}{2}mv^2 \\ m &= .02\text{kg} & .36\text{J} &= \frac{1}{2}(.02\text{kg})v^2 \\ v &=? & v &= \sqrt{(2)(.36)/(.02\text{kg})} \\ & & \mathbf{v} &= \mathbf{6\text{m/s}} \end{aligned}$$

$$\begin{aligned} 57. V &= 12\text{m/s} & \text{KE} &= \frac{1}{2}mv^2 \\ \text{KE} &= 3600\text{J} & 3600\text{J} &= \frac{1}{2}m(12\text{m/s})^2 \\ M &=? & m &= (2)3600\text{J}/(12\text{m/s}^2) \\ & & \mathbf{M} &= \mathbf{50\text{kg}} \end{aligned}$$

$$\begin{aligned} 58. M &= .5\text{kg} & \text{GPE} &= Mgh \\ h &= 2.5\text{m} & 20\text{J} &= (.5\text{kg})g(2.5\text{m}) \\ \text{GPE} &= 20\text{J} & g &= 20\text{J}/(.5\text{kg})(2.5\text{m}) \\ g &=? & \mathbf{g} &= \mathbf{16\text{m/s}^2} \end{aligned}$$

$$\begin{aligned} 59. h &=? & \text{GPE} &= mgh \\ \text{GPE} &= 2400\text{J} & 2400\text{J} &= (50\text{kg})(9.8)h \\ M &= 50\text{kg} & H &= 2400\text{J}/(9.8) (50\text{kg}) \\ & & \mathbf{H} &= \mathbf{4900\text{m}} \end{aligned}$$

$$\begin{aligned} 60. \text{EPE} &= (3 \cdot 10^{-3})\text{J} & \text{EPE} &= \frac{1}{2}Kx^2 \\ x &= .01 & 3 \cdot 10^{-3} \text{J} &= \frac{1}{2}k(.01)^2 \\ & & k &= 3 \cdot 10^{-3}/(9.8 \cdot 10^{-4}) \\ & & \mathbf{K} &= \mathbf{60 \text{ N/M}} \end{aligned}$$

$$61. V = 14\text{m/s} \quad \text{KE} = \frac{1}{2}mv^2$$

Conservation of Energy

Examples

$$\begin{aligned} \text{b) KE} &= \text{EPE} & \frac{1}{2}mv^2 &= \frac{1}{2}Kx^2 \\ k &= 250\text{N/M} & \sqrt{k(x^2)/m} &= v \\ x &= .05 & v &=? & \sqrt{250(.05^2)/.025} &= v \\ m &= .025 & & & & \\ & & \mathbf{v} &= \mathbf{5\text{m/s}} \end{aligned}$$

$$\begin{aligned} \text{e. EPE} &= \text{GPE} & \frac{1}{2}Kx^2 &= mgh \\ k &= 180\text{N/M} & k(x^2)/2gm &= h \\ m &= .004\text{kg} & h &=? & 180(.03^2)/2(9.8).004 &= h \\ x &= .03 & \mathbf{H} &= \mathbf{2.06\text{m}} \end{aligned}$$

$$\begin{aligned} 64. \text{A) GPE} &= \text{EPE} & \frac{1}{2}Kx^2 &= mgh \\ mgh &= \frac{1}{2}Kx^2 & k(x^2)/2gm &= h \\ m &= .002\text{kg} & 120(.04^2)/2(9.8).002 &= h \\ k &= 120\text{N/M} & & & & \\ x &= .04 & H &=? & \mathbf{H} &= \mathbf{4.89\text{m}} \end{aligned}$$

$$\begin{aligned} \text{B) KE} &= \text{EPE} & \frac{1}{2}mv^2 &= \frac{1}{2}Kx^2 \\ v &=? & \sqrt{k(x^2)/m} &= v \\ & & \sqrt{120(.04^2)/.002} &= v \\ & & \mathbf{v} &= \mathbf{9.8\text{m/s}} \end{aligned}$$

$$\begin{aligned} \text{b) KE} &= \text{GPE} & \frac{1}{2}mv^2 &= mgh \\ v &= 25\text{m/s} & (v^2)/2g &= H \\ g &= 9.8 \text{ m/ss} & (25^2)/2(9.8) &= H \\ & & \mathbf{H} &= \mathbf{31.9\text{m}} \end{aligned}$$

Class Work

$$\begin{aligned} \text{f. } W &= Fd & F &= mg \\ m &= 3\text{kg} & F &= 3(9.8) = 29.4\text{N} \\ d &= .75\text{m} & W &= (29.4)(.75) \\ & & W &= 22.05\text{J} \end{aligned}$$

Class Work Cont'd

$$\begin{aligned} \text{e. GPE} &= \text{KE} & mgh &= \frac{1}{2}mv^2 \\ H &= 134\text{m} & \sqrt{2gh} &= v \\ g &= 9.8 \text{ m/ss} & \sqrt{2(9.8)134} &= v \end{aligned}$$

$$v = \underline{51.24 \text{ m/s}}$$

2) $F = kx$ $EPE = GPE$ $1/2Kx^2 = mgh$
 $F = 10 \text{ N}$ $k(x^2)/2gm = h$
 $x = .05$ $200(.05^2)/2(9.8).002 = h$
 $F/x = k$
 $10/.05 = 200 \text{ N/M}$ $H = \underline{12.8 \text{ m}}$

1) $GPE = KE$ $1/2mv^2 = mgh$
 $v = 8 \text{ m/s}$ $(v^2)/2g = H$
 $g = 9.8 \text{ m/ss}$ $(8^2)/2(9.8) = H$
 $H = \underline{3.27 \text{ m}}$

1) $GPE = EPE$ $GPE = 1/2Kx^2$
 $GPE = 56 \text{ J}$ $\sqrt{2(56)/130} = x$
 $k = 130 \text{ N/M}$ $x = \underline{.93}$

1) A) $E_o + W = E_f$ $W = E_f - E_o$
 $m = 8.5 \cdot 10^5$ $v_f = 15$ $v_i = 10$
 $W = 8.5 \cdot 10^5 (.5)((15^2) - (10^2))$
 $W = \underline{5.3125 \cdot 10^7 \text{ J}}$

B) $E_o + W = E_f$ $W = E_f - E_o$
 $m = 8.5 \cdot 10^5$ $v_f = 20 \text{ m/s}$ $v_i = 15 \text{ m/s}$
 $W = 8.5 \cdot 10^5 (.5)((20^2) - (15^2))$
 $W = \underline{7.4375 \cdot 10^7 \text{ J}}$

C) $KE = KE = 1/2mv^2$
 $v = 20 \text{ m/s}$
 $KE = 1/2(8.5 \cdot 10^5)(20^2)$
 $KE = \underline{1.7 \cdot 10^8 \text{ J}}$

Homework

1) $KE = W$ $KE = 1/2mv^2$
 $m = 2000 \text{ kg}$ $KE = (1/2)(2000)(30^2)$
 $v = 30 \text{ m/s}$ $W = \underline{900,000}$

73. $KE = GPE$ $mgh = 1/2mv^2$
 $H = 2.7 \text{ m}$ $\sqrt{2gh} = v$
 $g = 9.8 \text{ m/ss}$ $\sqrt{2(9.8)2.7} = v$
 $v = \underline{7.27 \text{ m/s}}$

74. $KE = GPE$ $mgh = 1/2mv^2$
 $H = 125 \text{ m}$ $\sqrt{2gh} = v$
 $g = 9.8 \text{ m/ss}$ $\sqrt{2(9.8)125} = v$

$$v = \underline{49.49 \text{ m/s}}$$

$KE = 1/2mv^2$ $KE = W$ $m = 1500 \text{ kg}$
 $KE = (1/2)(1500)(20^2)$ $v = 20 \text{ m/s}$
 $W = \underline{300,000 \text{ J}}$

76. $KE = GPE$ $1/2mv^2 = mgh$
 $v = 190 \text{ m/s}$ $(190^2)/2g = H$
 $g = 9.8 \text{ m/ss}$ $(190^2)/2(9.8) = H$
 $H = \underline{1841.83 \text{ m}}$

77. $EPE = KE$ $1/2mv^2 = 1/2Kx^2$
 $k = 395 \text{ N/m}$ $\sqrt{k(x^2)/m} = v$
 $x = .005$ $v = ?$ $\sqrt{395(.005^2)/.5} = v$
 $m = .5 \text{ kg}$
 $v = \underline{.14 \text{ m/s}}$

A) $EPE = KE$ $1/2Kx^2 = mgh$
 $k = 220$ $k(x^2)/2gm = h$
 $x = .02$ $220(.02^2)/2(9.8)(.003) = h$
 $m = .003 \text{ kg}$ $H = \underline{1.5 \text{ m}}$

B) $EPE = KE$ $1/2Kx^2 = mgh$
 $x = .04$ $k(x^2)/2gm = h$
 $220(.04^2)/2(9.8)(.003) = h$
 $H = \underline{6 \text{ m}}$

Power Examples

$P = W/t$ $t = 1800 \text{ s}$ $W = 23 \cdot 10^3 \text{ J}$
 $P = (23 \cdot 10^3)/1800$

$$P = \underline{12.77 \text{ W}}$$

Power Examples Cont'd

$P = W/t$ $W = Pt$
 $t = 3.5(60) = 210 \text{ s}$
 $P = 15 \cdot 10^3 \text{ W}$
 $W = 15 \cdot 10^3(210)$
 $W = \underline{3.15 \cdot 10^6 \text{ J}}$

$P = W/t$ $t = W/P$
 $t = (300 \cdot 10^3)/(400)$

$$t=750s$$

$$P=Fv \quad P=(12)(3)$$

$$F=12N$$

$$v=3m/s \quad \underline{P=36W}$$

Class Work

$$P = W/t$$

$$W=3500 \cdot 10^3 \quad P = (3500 \cdot 10^3)/300$$

$$t=5(60)=300s \quad \underline{P=11,666.7W}$$

$$P=W/t \quad W=Fd$$

$$P=500W \quad Pt/d = F$$

$$t=12s \quad (500)(12)/96 =$$

$$d=96m \quad \underline{F=62.5N}$$

$$P = W/t$$

$$Pt = W \quad (250 \cdot 10^6 w)(468) = W$$

$$P=250 \cdot 10^6 \text{ MW}$$

$$t=468s \quad \underline{W=1.17 \cdot 10^{11} \text{ J}}$$

$$P = W/t$$

$$t = W/P \quad (278 \cdot 10^3)/(95 \cdot 10^3) = t$$

$$P = 95 \cdot 10^3 \text{ KW} \quad \underline{t=2.9s}$$

$$W=278 \cdot 10^3 \text{ KJ}$$

$$P=Fv$$

$$P=150 \cdot 10^3 \text{ kW} \quad P/v = F$$

$$v=28 \cdot 10^3 \text{ Km/s}$$

$$(150 \cdot 10^3)/(28 \cdot 10^3) = F$$

$$\underline{F=5.357N}$$

$$P=Fv$$

$$F=467 \cdot 10^3 \text{ kN}$$

$$v = 21.6 \cdot 10^3 \text{ Km/s}$$

$$P = (467 \cdot 10^3)(21.6 \cdot 10^3)$$

$$\underline{P=1.008 \cdot 10^{10} \text{ W}}$$

$$P=Fv$$

$$F = 7.35 \cdot 10^3 \text{ kW} \quad P/F=v$$

$$P=5.56 \cdot 10^3 \text{ kN}$$

$$v=(7.35 \cdot 10^3)/(5.56 \cdot 10^3)$$

$$\underline{v=1.32m/s}$$

Homework

87. $P = Fd/t$

$$t = Fd/P \quad m = 2000\text{kg}$$

$$F=mg = (2000)(9.8)=19600\text{N}$$

$$t = (19600)(28)/(13 \cdot 10^3)$$

$$\underline{t=42.2s}$$

$$P=mgd/t \quad P=50 \cdot 10^3 \quad t=(1.4)(60)=84$$

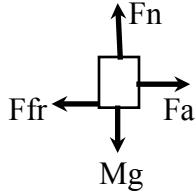
$$pt/dg=m \quad d=84\text{m} \quad g=9.8\text{m/ss}$$

$$(50 \cdot 10^3)(84)/(50)(9.8) = m$$

$$\underline{m=8571.42\text{kg}}$$

General Problems

92.a.



17. $\Sigma F = ma$
 $F_a - \mu mg = ma$
 $225 - (.3)(46)(10) = 46a$
 $a = \underline{2.54 \text{ m/s}^2}$

101. $x = x_o + v_o t + \frac{1}{2} a t^2$
 $x = \frac{1}{2} a t^2$
 $\frac{1}{2} (2.54)(10)^2$
 $x = \underline{127 \text{ m}}$

d. $v = v_o + at$
 $(2.54)(10)$
 $v = \underline{25.4 \text{ m/s}}$

i. $KE = \frac{1}{2} m v^2$
 $\frac{1}{2} (46)(25.4)^2$
 $KE = \underline{14838.68 \text{ J}}$

i. A) $W = FD$
 $(255)(127)$
 $W = \underline{32835 \text{ J}}$

c) $W = \mu mg D$ $F = \mu mg = 138$
 $(138)(127) =$
 $W = \underline{17526 \text{ J}}$

f. $W = 0 \text{ J}$

c) $W = 0 \text{ J}$

g. $W = Fd$
 $F = 255 - 138 = 117$
 $(117)(138) = \underline{16146 \text{ J}}$

g. they are about the same

3) a. $\Sigma F = ma$
 $F = \mu mg$
 $F = (.15)(50)(9.8)$
 $F = \underline{73.5 \text{ N}}$

2) $W = FD$
 $(73.5)(7.5)$
 $W = \underline{551.25 \text{ J}}$

2) Same as part b (551.25J)

- 2) 0
- 2) 0
- 3) 0
- 4) 0

94a. $GPE = MGH$
 $(.25)(10)(8)$
 $GPE = \underline{2 \text{ J}}$

$GPE = KE$
 $GPE = \frac{1}{2} m v^2$
 $v = \sqrt{(2GPE/m)}$
 $\sqrt{(4/.25)}$
 $v = \underline{4 \text{ m/s}}$

$GPE_1 = KE + GPE_2$
 $\sqrt{(2GPE_1 - 2GPE_2)/m}$
 $\sqrt{(4 - 1.5)/.25}$
 $v = \underline{3.16 \text{ m/s}}$

$F = m v^2 / r$
 $F = (.25)(4^2) / .15$
 $F = \underline{26.66 \text{ N}}$

$F = m v^2 / r$
 $F = (.25)(3.16^2) / .3$
 $F = \underline{8.321 \text{ N}}$

General Problems (Cont'd)

95.

a. EPE

$$\begin{aligned} \frac{1}{2}Kx^2 &= \text{EPE} \\ \frac{1}{2}(65)(.035^2) &= \underline{\text{EPE} = .05\text{J}} \end{aligned}$$

It alternates between EPE and KE.

$$\begin{aligned} \text{EPE} &= \text{KE} \\ .05 &= \frac{1}{2}mv^2 \\ \sqrt{2}(.05)/.8 &= \underline{.35\text{m/s} = \text{KE } v \text{ max}} \end{aligned}$$

96.

a. $W = Fd$ $W = \text{area triangle}$

$$.5(10)(20) = \underline{100\text{J} = W}$$

$$\text{KE} = W \quad \underline{W = 100\text{J}}$$

$$\begin{aligned} \text{KE} &= \frac{1}{2}mv^2 \\ \sqrt{2}(100)/2.5 &= \underline{8.94 \text{ m/s} = v} \end{aligned}$$

$$\begin{aligned} \underline{300\text{J} = W} &= 100 + 200 \\ 100 &= (.5)(10)(20) \\ 200 &= (.5)(20)(20) \end{aligned}$$

$$\text{KE} = W \quad \underline{300\text{J} = \text{KE}}$$

$$\begin{aligned} \text{KE} &= \frac{1}{2}mv^2 \\ \sqrt{2}(300)/2.5 &= \underline{15.49 \text{ m/s} = v} \end{aligned}$$

97.

$$F = ma \quad F/m = a \quad 4/2 = \underline{2\text{m/ss} = a}$$

$$\begin{aligned} \text{KE} &= \frac{1}{2}mv^2 \\ \sqrt{2}(48)/2 &= 6.92/2 = \underline{3.46\text{s} = t} \end{aligned}$$

c. $w = Fd$ $W = \text{area}$ $W = (12)(4) = \underline{48\text{J} = W}$

d. $\text{KE} = \frac{1}{2}mv^2$ $\sqrt{2}(48)/2 = \underline{6.92\text{m/s} = v}$

e. $\text{KE} = \frac{1}{2}mv^2 = (1/2)(2)v^2 = \sqrt{64} = \underline{8\text{m/s} = v}$

a. $\text{GPE} = mgh$
 $(1.5)(9.8)(2.5) = \underline{36.75\text{J} = \text{GPE}}$

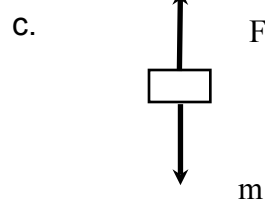
b. $\text{GPE}_i = \text{KE}_{\text{top}} + \text{GPE}_{\text{top}}$
 $\text{GPE}_i - \text{GPE}_{\text{top}} = \text{KE}_{\text{top}}$ (solve for v in KE top)
 $\sqrt{2}(36.75 - ((10)(1.8)(1.5)))/1.5 = \underline{3.7\text{m/s} = v}$

c. $\text{KE} = \frac{1}{2}mv^2$
 $\sqrt{2}(36.75)/1.5 = \underline{7\text{m/s} = v}$

d. $\text{EPE} = \text{KE}$ $\text{KE} = 36.75$
 $\frac{1}{2}Kx^2 = 36.75$
 $\sqrt{2} 36.75/90 = \underline{.9\text{m} = x}$

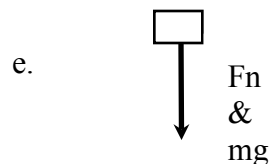
a. $\text{GPE} = mgh$
 $(500)(9.8)(70) = \underline{343,000\text{J} = W}$

b. $\text{GPE} = \text{KE}$
 $\sqrt{2}(343000)/500 = \underline{37.04 \text{ m/s} = v}$



d. $\text{GPE}_i - \text{GPE}_{\text{top}} = \text{KE}_{\text{top}}$ (solve for v in KE top)

$$\sqrt{2}(343000 - ((9.8)(30)(500)))/500 = \underline{28\text{m/s} = v}$$



f. The speeds at A and C would be less.