

## Electromagnetic Induction

### Introduction

Earlier, we learned how an electric current produces a magnetic field and how a current carrying wire feels a force due to an external magnetic field, clearly linking electricity and magnetism; those discoveries were made in 1820-21. It made sense to people that if electric currents (which are the result of an electric field) produce magnetic fields, then magnetic fields should be able to create electric fields.

The answer to whether, and how, this could be done proved elusive for about a decade, until Joseph Henry (1797–1878) and Michael Faraday (1791–1867) each independently discovered the secret. What made it so hard to figure out was that there is no such effect in the static case. That is, placing a magnet near a wire does not create an electric field, and a resultant current. *Changing* magnetic fields would be found to induce electric fields, not static magnetic fields.

This discovery would prove to have a huge impact on human history. Up until that discovery, electricity could only be produced through the use of a battery: a relatively complicated process involving the construction of battery cells which would have to be frequently replenished with chemicals. This new discovery allowed for the conversion of mechanical energy directly into electrical energy. It's the source of 99% of all the electric power we use today and allowed modern civilization to emerge.

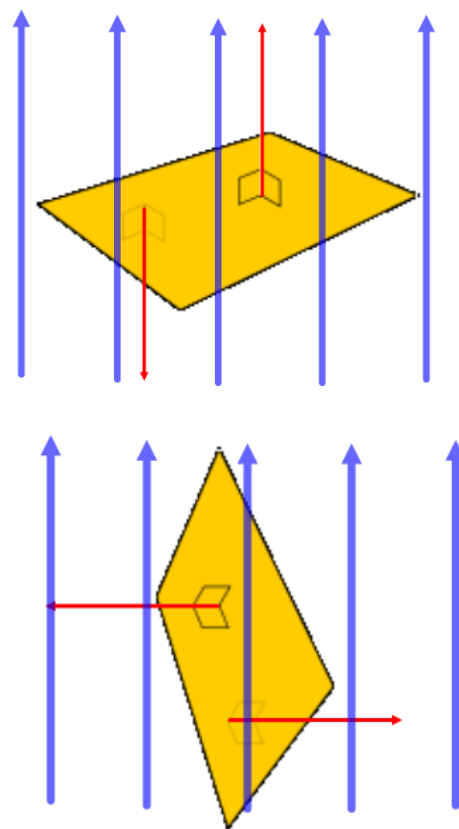
It's said that one day the Prime Minister of the United Kingdom observed a small experiment that Michael Faraday had set up to show how this conversion of mechanical to electrical energy took place. The Prime Minister asked of the new discovery, "What good is it?" Faraday is said to have replied, "What good is a new-born baby?" Today all the electrical devices you see around you were made useful by this discovery by Henry and Faraday. The very skyline of every modern city was made possible by elevators which rely on this discovery. Every time you see anything that directly or indirectly relies on electricity you should know that it was made possible by the phenomenon you're about to learn about.

### Magnetic Flux

In order to understand Magnetic Induction, you must first understand magnetic flux. Magnetic flux is a measure of the number of magnetic field lines that are passing through an imaginary surface: the magnetic flux through that surface. That will depend on the size of the surface, the strength of the magnetic field and the orientation of the surface to the magnetic field. In this course, we will only consider two possible orientations, perpendicular and parallel. If the surface is perpendicular to the field, all the field lines present will pass through it. If the surface is parallel to the B-field, then the field lines will pass along, but not through, the surface.

In the top drawing, the surface is perpendicular to the field (the small arrows up and down from the surface are normal to the surface, so when they are parallel to the field, the field is perpendicular to the surface) so all the field lines go through the surface. To increase the magnetic flux, we would have to increase the number of field lines (increase the B-field) or increase the size of the surface. If we rotate the surface so that it is parallel to the B-field, as in the bottom figure, no field lines will pass through it.

The symbol for flux is  $\Phi$  (the Greek letter "phi") and so the symbol for magnetic flux is  $\Phi_B$ . In the case that the surface and field are perpendicular, the magnetic flux is proportional to the strength of the B-field and the area of the surface:  $\Phi_B = BA$ . When the field and surface are parallel,  $\Phi_B = 0$ .



$$\Phi_B = BA$$

Where  $\perp$  is the symbol for “perpendicular”, B is the magnitude of the magnetic field perpendicular to the surface, and A is the area of the surface. The unit of magnetic flux is the Weber (Wb). If the magnetic field is measured in Teslas (T) and the area is measured in square meters (m<sup>2</sup>), then 1Wb = 1T·m<sup>2</sup>.

$$1\text{Wb} = 1\text{T}\cdot\text{m}^2$$

*Example 1: A 4.5 m<sup>2</sup> surface is in a 0.60T magnetic field. What is the magnitude of the magnetic flux through the surface if it is (a) perpendicular to the field? (b) parallel to the field?*

- Solution:
- a)  $\Phi_B = BA$   
 $\Phi_B = (0.60\text{T})(4.5\text{ m}^2)$   
 $\Phi_B = 2.7\text{ Wb}$
- b) zero, there is no flux if the field and surface are parallel

*Example 2: A circular surface of radius 4.0m is in a 0.40T magnetic field. What is the magnitude of the magnetic flux through the surface if the surface is (a) perpendicular to the field? (b) parallel to the field?*

- Solution:
- a)  $\Phi_B = BA$   
 $\Phi_B = B(\pi r^2)$   
 $\Phi_B = B\pi r^2$   
 $\Phi_B = (0.40\text{T})(3.14)(4.0\text{m})^2$   
 $\Phi_B = 20\text{ Wb}$
- b) zero, there is no flux if the field and surface are parallel

### Faraday's Law of Induction

Faraday and Henry discovered that changing the flux through a loop of wire induces an electric field in it that results in the flow of electrical current. The amount of current depends on how quickly the flux changes and the number of coils of wire in the loop.

$$\mathcal{E} = -N \frac{\Delta\Phi_B}{\Delta t}$$

- **E** represents the induced electromotive force and is measured in volts,
- N stands for the number of turns of wire (this is a number and has no units)
- $\Delta\Phi_B$  is the change in the flux through the coil of wire ( $\Delta\Phi_B = \Phi_{B\text{final}} - \Phi_{B\text{initial}}$ ) and is measured in Webers
- $\Delta t$  is the time interval during which this change of flux occurs and is measured in seconds.
- The negative sign in front of “N” will be discussed later, under Lenz’s Law, and has to do with the direction of the induced voltage and resulting current

Some quick observations:

- The larger the number of turns of wire, the large the voltage that will be induced
- The greater the change in flux, the greater the induced voltage
- The shorter the time interval (the faster the change) the greater the induced voltage

*Example 3: A 1.2 m<sup>2</sup> coil of wire has 8 loops and is perpendicular to a 0.50T magnetic field. What is the induced voltage if (a) The B-field is reduced to zero in 0.20s, (b) the coil is rotated to be parallel to the field in 0.20s, (c) the field triples in 0.40s, (d) the area of the coil triples in 0.40s.*

Solution:

$$a) \quad E = \frac{-N\Delta\phi_B}{\Delta t}$$

$$E = \frac{-N\Delta(BA)}{\Delta t}$$

$$E = \frac{-NA\Delta B}{\Delta t}$$

$$E = \frac{-NA(\phi_{Bfinal} - \phi_{Binitial})}{\Delta t}$$

$$E = \frac{-8(1.2m^2)(0 - .5T)}{.2s}$$

$$E = \frac{(9.6m^2)(-.5T)}{.2s}$$

$$E = 24V$$

$$\Phi_B = B A$$

Since A is constant, it can move in front of the  $\Delta$

$$\Delta\Phi_B = \Phi_{Bfinal} - \Phi_{Binitial}$$

- b) When the coil becomes parallel to the B-field the flux through it is zero, as it did in (a). All other conditions are the same, so we get the same answer as we did for (a), 20 V.

$$c) \quad E = \frac{-N\Delta\phi_B}{\Delta t}$$

$$E = \frac{-N\Delta(BA)}{\Delta t}$$

$$E = \frac{-NA\Delta B}{\Delta t}$$

$$E = \frac{-NA(\phi_{Bfinal} - \phi_{Binitial})}{\Delta t}$$

$$E = \frac{-8(1.2m^2)(1.5 - .5T)}{.4s}$$

$$E = -24V$$

$$\Phi_B = BA$$

Since A is constant, it can move in front of the  $\Delta$

$$\Delta\Phi_B = \Phi_{Bfinal} - \Phi_{Binitial}$$

$$d) \quad E = \frac{-N\Delta\phi_B}{\Delta t}$$

$$E = \frac{-N\Delta(BA)}{\Delta t}$$

$$E = \frac{-NB\Delta A}{\Delta t}$$

$$E = \frac{-NB(A_{final} - A_{initial})}{\Delta t}$$

$$E = \frac{-8(.5T)(3.6m^2 - 1.2m^2)}{.4s}$$

$$E = \frac{(-4T)(2.4m^2)}{.4s}$$

$$E = -24V$$

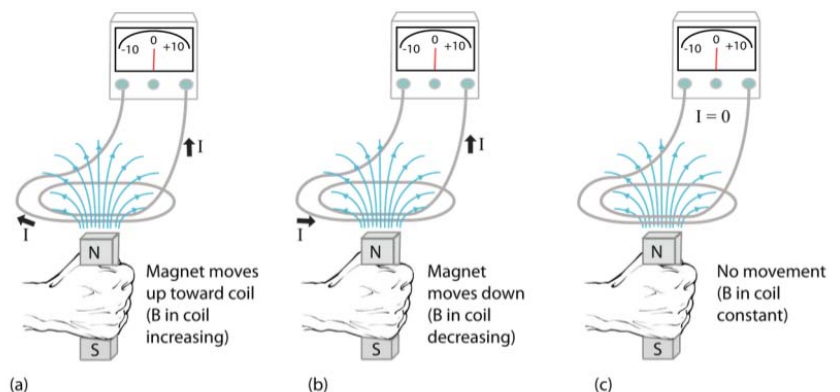
$$\Phi_B = BA$$

Since B is constant, it can move in front of the  $\Delta$

$$\Delta A = A_{final} - A_{initial}$$

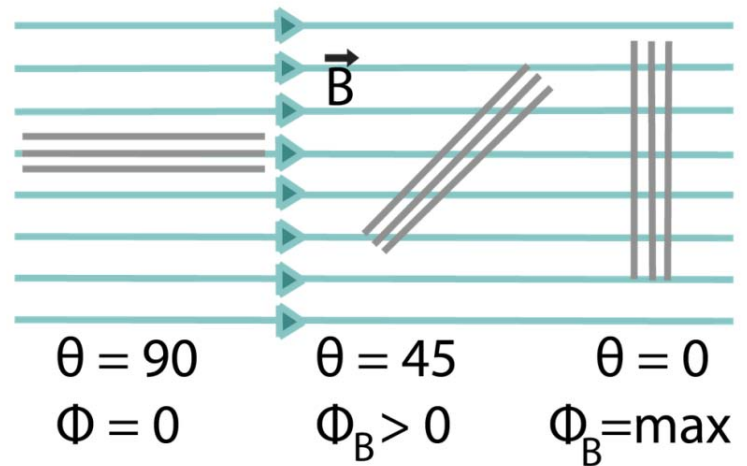
Note that in (c) and (d), tripling the area had the same effect as tripling the magnetic field, as we should have expected.

When the flux through a coil of wire is due to a nearby bar magnet it's important to picture the field lines emerging from the north pole of the magnet and returning to the south pole. As you can see in the below diagram, as the north pole of a magnet approaches a coil of wire, more of its field lines will go through the coil. The same is true if the south pole approaches the coil, but the field, and flux, will be in the opposite direction.



You can also see that if the magnet is held stationary, there will be a constant flux, which means no induced voltage. Only if the magnet is moved so that the number of field lines through the coil changes will a voltage be induced; that happens when you pull the magnet away or push it towards the coil.

Another way to induce an electric voltage, and current, is by rotating a coil of wire in a fixed magnetic field; that's how 99% of our electricity is produced. As you can see here, as the coil rotates in the field the flux will go from zero, to a maximum. As it continues to rotate the flux will decrease to zero again and then reach a maximum in the opposite direction. Since the flux is constantly changing, so is the induced voltage and current. This process results in an alternating voltage, and a resulting alternating current (AC) in the coil of wire. Imagine connecting a large coil of wire (large Area) in a strong magnetic field (large B-field) and then connecting that coil to something that keeps it rotating as shown in this diagram. That can be water flowing past paddlewheels, or a turbine, connected to the coil to get hydroelectricity; a steam turbine driven by the heat from burning gas or coal; turbines driven from steam generated by the heat of nuclear fission; wind driving windmills; etc. Virtually all the electric power in the world comes from rotating coils in magnetic fields; the only other commercially viable source is solar power.



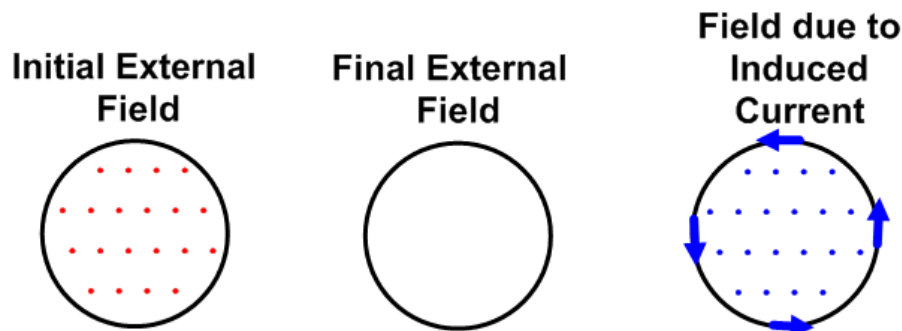
### Lenz's Law

Faraday's Law gives us a way of calculating the magnitude of an induced EMF, and from that we can calculate the magnitude of an induced current. But what is the direction of that current? The basic idea of how to determine that is seen in the "-" sign in the Faraday's Law. That tells us that the EMF will oppose the change in the magnetic flux, but how to interpret that?

Lenz's Law answers that question in a very general way: **the EMF induced by a change in the magnetic flux through a coil of wire will be such that it generates a current in the coil which creates a B-field that opposes that change in flux within the coil.** It's important to keep in mind that this relates only to the flux within the coil, not the field outside the coil, which is irrelevant to this.

*Example 4. Imagine a coil of wire lying flat on a horizontal table top. A magnetic field is directed upwards through the entire table top, including through the coil. What would be the direction of the induced current if the magnetic field were suddenly turned off?*

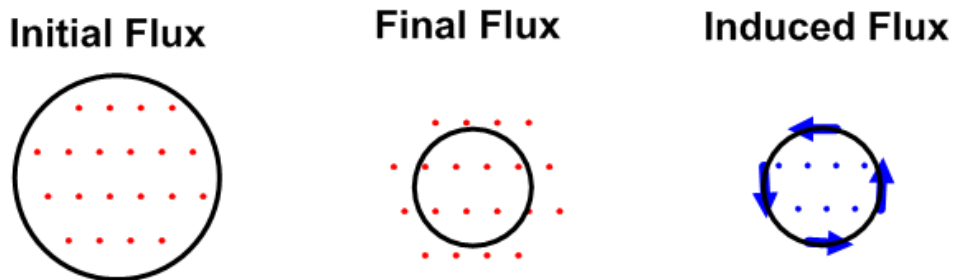
Solving problems of this sort is best done with a series of drawings: first, draw the initial situation; then draw the situation after the external effect; finally, compare those first two drawings and make a drawing of the field which would be required to go back to the initial drawing. Then, draw the direction of current in the coil of wire that would create the needed field **within** the coil. In this case,



the arrows drawn in the last drawing illustrate that a counterclockwise (CCW) current would be induced.

*Example 5. Imagine a coil of wire lying flat on a horizontal table top. A magnetic field is directed upwards through the entire table top, including through the coil. What would be the direction of the induced current if the coil was to shrink to half its size?*

In this case, because the coil becomes smaller, less external B-field lines will pass through it. Those must be replaced by the field created by the induced current. Keep in mind that the external and induced field lines together (in the second and third drawings) add together to replicate the initial flux (the first drawing).



The arrows drawn in the last drawing illustrate that a counterclockwise (CCW) current would be induced.

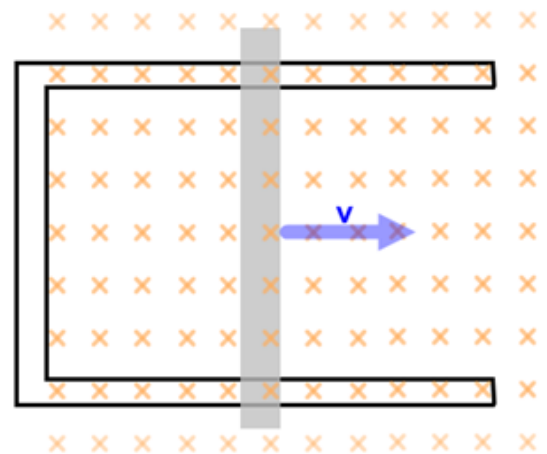
There are more general implications of Lenz's Law, which can be used to draw conclusions about situations without getting into the specifics of each case. For example, if you try to pull a coil of wire out of a magnetic field, Lenz's Law tells us that a current will be induced in the coil that will create a field, and resulting force, that opposes your effort to remove the coil: it will oppose your attempt to change the flux through the coil. Thus a force will be generated that is opposed to the force you are applying. This is the operating principle of electromagnetic brakes for cars, etc. You can figure out the direction of the current, but the basic idea, that the external force you apply to change the flux through the coil will be opposed, is valid even before looking into the mechanism.

In fact, Lenz's Law can be shown to emerge from the principle of Conservation of Energy. The system always tries to maintain its current state; external forces will be opposed by internal forces generated by the system in order to maintain that state.

### **Induced EMF in a Moving Conductor**

Let's look at a particular application of Lenz's Law and then see how that result can be derived independently and then made more general. We showed in Example 5 that if a coil that is perpendicular to a uniform magnetic field becomes smaller, a current will be induced. Let's look at another example of that. In this picture, a metal rod slides, without friction, such that the area enclosed by the four conducting sides, comprised of the three fixed sides and the moving rod, expands in a uniform perpendicular B-field. We determine the magnitude and direction of the induced current in the same manner as we did above.

The flux through the closed "coil" comprised of the three fixed and one moving side is given by  $\Phi_B = BA$ , where B is the constant external field and A is the area enclosed by the four conductors at any moment. The change in flux is given by  $\Delta\Phi_B = \Delta(BA)$ . Since B is not changing, but A is, we can rewrite this as  $\Delta\Phi_B = B\Delta A$ . If we call the distance between the two fixed rods "l" and the distance from the moving rod to the fixed rod parallel to it "w" this becomes  $\Delta\Phi_B = B\Delta(lw)$ . Once again, since l is fixed, we can move it in front of the  $\Delta$  operator and get  $\Delta\Phi_B = Bl\Delta w$ , which becomes  $\Delta\Phi_B = Blv\Delta t$ . Inserting this into Faraday's Law and simplifying gives the magnitude of the induced EMF.



$$E = \frac{-N\Delta\phi_B}{\Delta t}$$

$$E = -1 \frac{(Blv\Delta t)}{\Delta t}$$

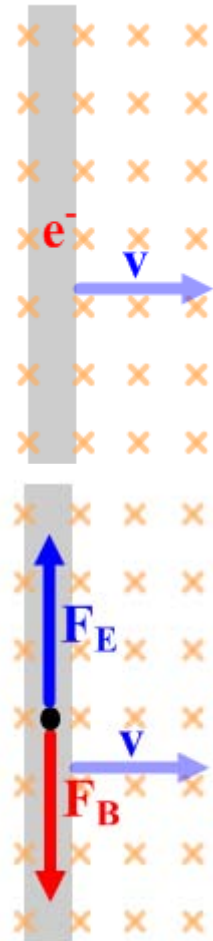
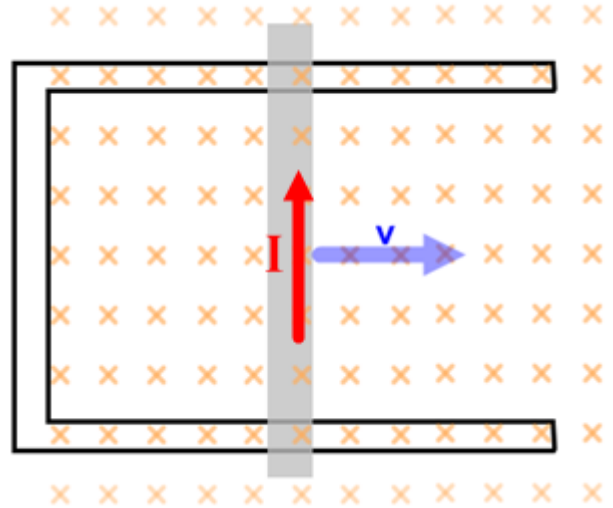
$E = Blv$  (ignoring the negative sign since we're just solving for magnitude, which is always positive)

Three factors will determine the magnitude of the voltage induced across the rod:

- the strength of the perpendicular B-field
- the length of the rod
- the velocity of the rod

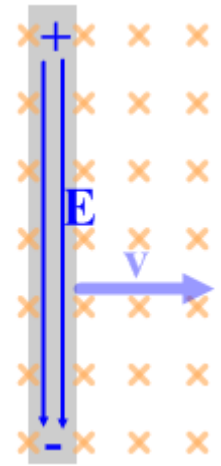
Now let's determine the direction of the induced current that results from that induced EMF.

The increasing amount of magnetic field, into the page, that is enclosed by the coil due to the rod being pulled to the right by an external force, will be opposed, due to Lenz's Law, by an induced current that will create a B-field that will be out of the page, canceling out that increasing flux. In order to create a field out of the page within the coil that will be out of the page, the induced current must be CCW. That means that a current will be induced that will run through the moving rod from the bottom to the top of the page.



This current can be thought of as being due to the growing size of the area enclosed by the four conductors (as we did above) or due to an induced EMF within the moving rod, ignoring the other conductors. Let's see if we get the same result.

Since the rod is a conductor, electrons,  $e^-$ , are free to move within it; let's first look at the magnetic force that an electron would experience due to its velocity perpendicular to an external magnetic field. Using the right hand rule, and reversing our result since this is a negative charge (or the left hand rule, if you prefer), we see that the magnetic force would drive electrons towards the bottom of the page. (If this were part of a complete circuit, as it was above, electrons would travel around the circuit with the effect of creating a conventional current in the rod towards the top of the page, as we found before.) However, since the rod, in this case, is not connected to a circuit, electrons will accumulate in the rod towards the bottom of the page, and a shortage of electrons will develop in the rod towards the top of the page. The result will be an electric field towards the bottom of the page.



Electrons won't continue to move to the bottom of the page indefinitely because an increasingly large electric field will push them towards the top of the page (remember, since electrons are negative they feel a force in the opposite direction of the E-field). We can do a free body diagram of the magnetic and electric forces acting on the electron and use that to determine the potential difference, EMF, along the length of the rod.

In this diagram we see that when equilibrium is achieved, there is no net force on the electrons in the rod, the induced electric field creates an electric force that is equal and opposite to the magnetic force on the electrons due to their motion in the magnetic field. Applying Newton's Second Law:

$$\mathcal{E}F = ma$$

$$F_E - F_B = 0$$

$$qE - qvB = 0$$

$$E = vB$$

$$\frac{V}{l} = vB$$

$$V = Bvl$$

$$\mathbf{E} = Blv$$

from the free body diagram

using the formulas for  $F_E$  and  $F_B$

adding  $qvB$  and canceling  $q$  from both terms

Using  $V = Ed$  to replace  $E$  with  $V$ , for electric potential

Solving for  $V$ , the electric potential difference

replacing  $V$  with the EMF, since that is the source of  $V$  here

We get the same result for the induced voltage in a rod as we got before using Faraday's Law for the case where the rod is one side of a complete circuit. In this case, we clearly see that an E-field is being created by the motion of a charged rod through a B-field.

*Example 6: A Boeing 777 airplane has a wingspan of about 68-m and flies at speeds up to about 250 m/s. Earth's magnetic field is up to about  $6 \times 10^{-5} T$ . What is the maximum potential difference that could be expected between its wingtips?*

$$\mathbf{E} = Blv$$

$$\mathbf{E} = (6 \times 10^{-5} T)(68m)(250m/s)$$

$$\mathbf{E} = 1.0 V$$