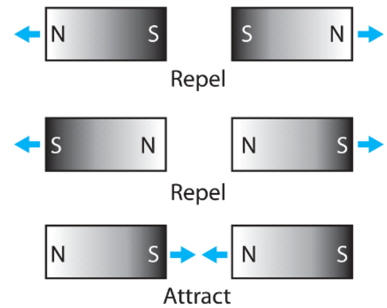
Magnetism

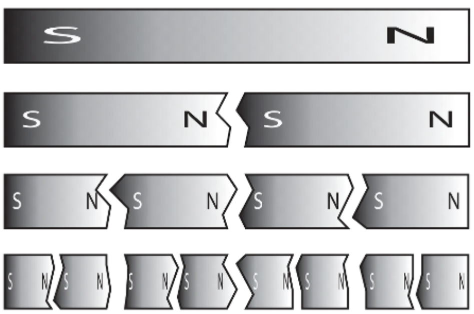
Introduction

Just as static electricity was known in ancient times, so was magnetism. In fact, the term "magnetism" comes from the rocks that were found in the ancient Greek province of Magnesia 4000 years ago; Lucretius, a Roman poet and philosopher, wrote about magnets more than 2000 years ago. Magnetic materials were considered even more special than electric materials since they came out of the ground with their force of attraction, while amber had to be rubbed to develop its static electric force.

It was also noted very early in human history that if magnetic rock was shaped into the form of a needled and floated on a surface of water, it always pointed in the same direction. We now call that direction "magnetic north." This was used by Chinese mariners more than 4000 years ago to navigate. Because this direction was the same regardless of your location and which direction you were traveling, it was always possible to know which way you were sailing.

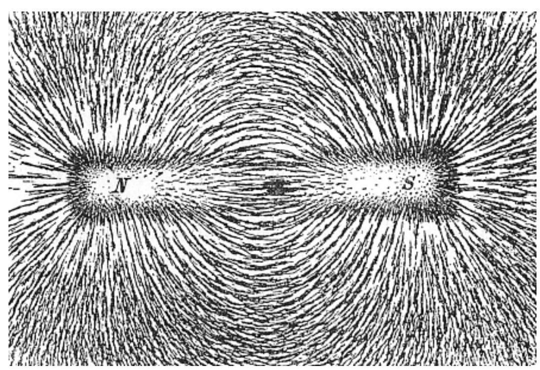


There is an important similarity between electricity and magnetism. Electric charge can be either positive or negative; like charges repel and unlike charges attract. Magnets have two poles, called "north" and "south"; like poles repel and unlike poles attract.



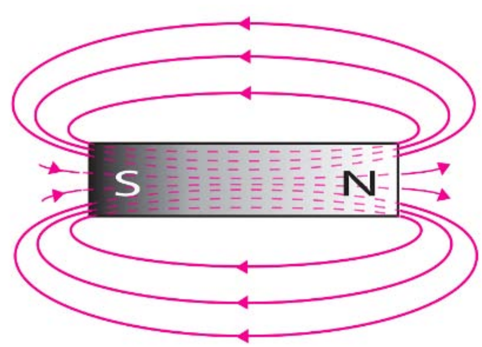
But there is a key difference as well; while it is possible to separate positive and negative electric charges (that's what happens when you rub a plastic rod with cloth), it is impossible to separate north and south poles. In fact, a north pole can only exist in the presence of an exactly equal south pole: "there are no magnetic monopoles." If a magnet is cut in half, the result is not the separation of the north and south poles, it's the creation of more equal and opposite poles.

Magnetic Field Lines



The force between magnets occurs without the magnets touching, it occurs at a distance. A good way to visualize how this works is to think of each magnet creating a magnetic field. The magnetic field from one magnet interacts with the magnetic field of another; as these fields interact they generate a force between the magnets so that each magnet tries to orient itself to line up with the magnetic field of the other. Let's see how that works.

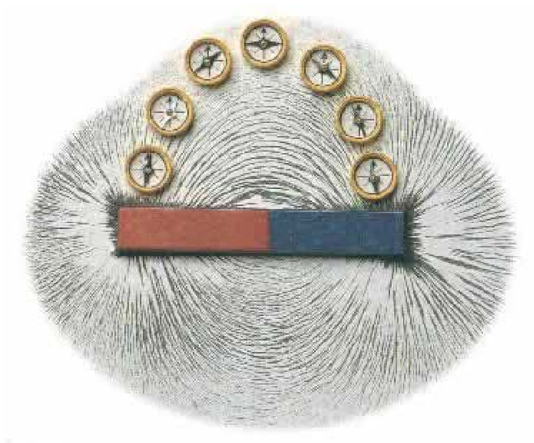
Iron filings can act like small magnets. When a magnet is placed under a piece of paper and iron filings are sprinkled onto it, the filings rotate to line up with the field of the magnet, just like any magnet would, making the field visible. That's exactly what you see in this photograph: .



The field seems to come out of one pole and return to the opposite pole, forming a complete loop when the magnet itself is included. (This is illustrated in this drawing from Giancoli Physics.) While the loop is clear, it's arbitrary to decide which side of the magnet the field comes out of and which side the field reenters; by convention we take it that the field leaves the north pole and reenters the south pole.

We know that opposite poles attract and like poles repel, the south pole of the filings rotate to be closer to the north pole of the magnet, while the south poles orient themselves to be farther away. This can be seen in the below photograph, which includes iron filings as well as small compasses (http://www.school-for-champions.com/science/images/magnetic_detection-iron_filings.jpg.)

Each compass needle or iron filing is a magnet; each needle up parallel to the magnetic field at that location in space with its north pole pointing along the field lines in the direction of the magnet's south pole. It does this because it minimizes the energy of the system to have the magnetic fields going in the same direction. Any magnet will rotate in an external magnetic field so that its field aligns with the external field. The field leaving its north pole points in the direction of the field leaving the north pole of the magnet creating the external field.

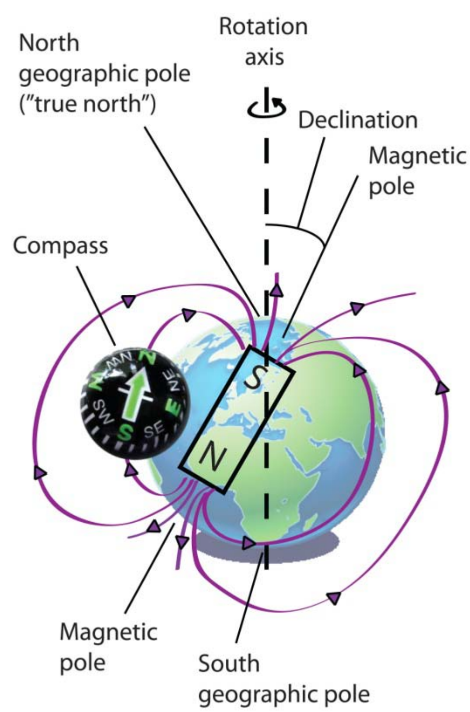


If you have two magnets, it's fair to ask which one creates the external field and which one experiences the magnetic force that makes it align with that external field. The answer is that each magnet both creates an external field and responds to the net external field due to the other magnet (or the net magnetic field due to multiple magnets). The magnetic force that acts on one magnet is equal and opposite to the force that magnet exerts on the other. This is just another example of Newton's Third Law; each magnet creates a force on the other, and responds to that same force acting on it.

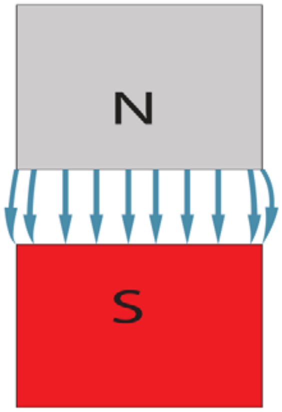
Magnetic fields are just as real as anything in nature, even though they can't be seen without the help of compasses, other magnets or iron filings. In much of this chapter, we'll be discussing the effect of magnetic fields in nature, regardless of their source.

Earth's Magnetic Field

Earth has a molten iron core that's rotating due to the overall rotation of our planet. That rotating core creates a giant magnetic field that emerges near one of its poles (as defined by Earth's axis of rotation) and reenters near the opposite pole. That field is vitally important to life on Earth as it deflects, in a manner we'll be explaining soon, cosmic rays and channels them away from the middle latitudes; few of them reach the surface of the planet at all, and those few strike near the poles, not in the middle latitudes. (Some believe that for a planet to evolve life as we know it requires that it have a magnetic field to offer this kind of protection.)



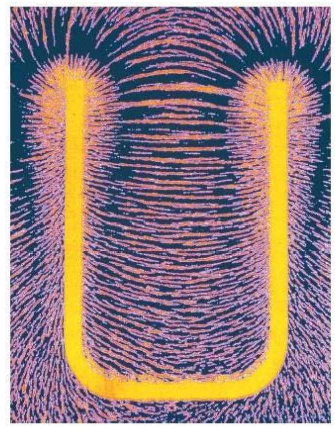
We know that the north pole of a compass points in the direction we call North. And we know that one of the geographic poles of Earth will be close to a magnetic north pole, and the other will be close to a magnetic south pole, but which is which. While we would think that the geographic north pole of Earth would be a magnetic north pole, we also know that can't be the case; if it were, the north pole of a compass would point away from it, not towards it. In reality, the geographic north pole of Earth (based on its rotational axis) is very near the Earth's magnetic south pole. The magnetic south pole of Earth is near its geographic north pole.



Uniform Magnetic Fields

The magnetic field around a bar magnet varies in strength and direction, as you can see by looking at the photographs above. As was the case with electric fields, the direction of the magnetic field is signified by the direction of the arrows used to depict it, and its strength is indicated by the density of the magnetic field lines. The closer together the field lines are drawn, the stronger the field at that location.

In the case of a uniform magnetic field, the field lines are parallel and equally spaced (the same is true for the electric and gravitational fields we learned about earlier). By convention, fields lines always leave the north pole and enter the south pole of a magnet. To create a uniform field, a small gap is created between two relatively large opposite magnetic poles; the field is then constant between the poles. In practice, there is some bending of the field lines towards the edges, but that can be neglected if the gap is made very small compared to the size of the poles.



One way to create a somewhat uniform magnetic field is with a "horseshoe" magnet. One side of the horseshoe is magnetized to be a north pole and the other side a south pole. In the gap between the poles the B-field leaves the north and enters the south pole. If the poles are large and relatively close together the field can be considered uniform between them.

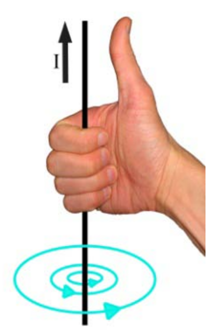
This photograph is of iron filings near a horseshoe magnet. Note that directly between the two sides of the horseshoe the field is relatively uniform. It can't be determined from this photograph which of the poles is north and which is south, but we know that they have equal and opposite magnetic polarity (http://www.physics.ohio-state.ed /~yuri/112/horse_shoe_magnet.jpg). When working with any magnet, always draw the field from the north to the south pole.

Electric Currents Produce Magnetic Fields

One of the most amazing discoveries of the 19th century was the connection between electricity and magnetism. Previously, these were considered separate phenomena. For thousands of years people had experienced static electricity; similarly for those thousands of years they had experienced magnets. The idea that they were connected didn't emerge until a lucky coincidence in 1820. Hans Christian Ortsted was preparing a physics lecture about electricity. He had set up an electric circuit with a battery and a switch. When he closed the switch, and current began to run through the wire, he noticed that a compass that was near the wire rotated, as if it was trying to line up with a magnetic field, just as a compass rotates when it is near a bar magnet as shown in the photograph above. There was only one explanation: the electric current was creating a magnetic field.

Direction of the Induced Magnetic Field

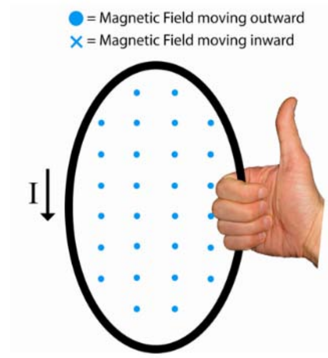
Experiments showed that the direction of the magnetic field created by an electric current is circular: it flows as concentric circles around the wire. Its direction is described by using the first of two "right hand rules" we'll be describing. In this case, to find the direction of the magnetic field surrounding a current carrying wire, point your right thumb in the direction of the current; the magnetic field goes in a circle described by the direction of the curl of your right fingers

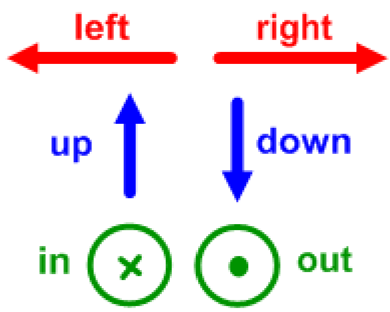


Given a circular loop of wire, the field can be described as going in one side of the loop and out the other. That's because no matter where you grasp the loop with your right hand, so that your thumb is in the direction of the current, your fingers will enter the loop on one side, pass through it, and exit the loop on the other; the magnetic field does the same.

<u>Describing Direction in a 3-D World</u>

So far in this book we've dealt with problems that could be described and solved with 2-dimensional (2-D) approaches. But as we can already begin to see, and as will quickly become more evident, magnetism is a fundamentally 3-dimensional (3-D) effect. So we need some good tools to describe the six directions of a 3-D world. These tools will be used in this chapter to describe the direction of current, force and field in the six possible directions.

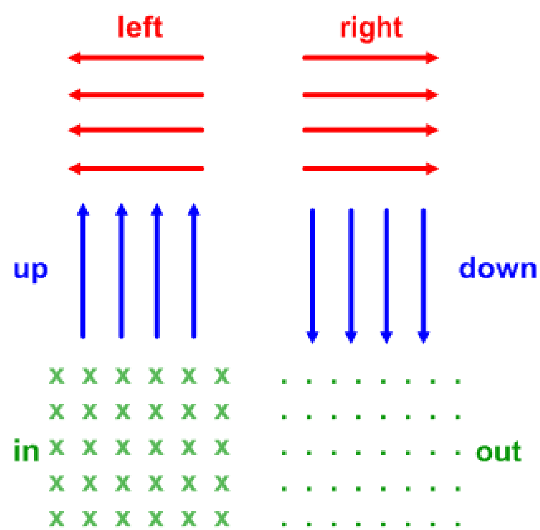




While the first four direction indicators are obvious, the last two are new and require some explanation. Think of the current, force or field, being described as an arrow, with a point on one side (in the direction its going) and crossed feathers on the other.

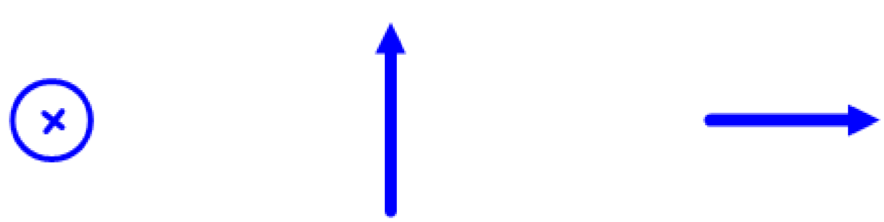
If the arrow is flying towards you, out of the page, you'll just see its point, which will look like a dot. If the arrow is flying away from you, you'll see the crossed feathers, or an "x". So if the field is going into the page you'll see an "x" and if the field is coming out of the page, towards you, you'll see a dot.

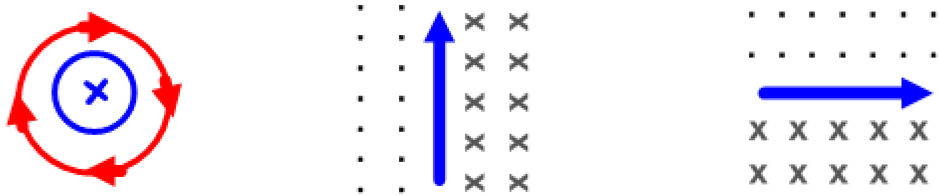
While these symbols work well to describe the directions of current and force, one additional adaption is needed to describe the direction of a magnetic field. That's because fields occupy a large volume of space, while current and force can be thought of as lines. Here's how the six possible magnetic field directions are drawn using the above convention.



Using these tools, and the right hand rule, you can draw and solve problems involving the magnetic field created by an electric current. Let's draw a current in flowing in each of the three possible dimensions so we get used to how to use these symbols.

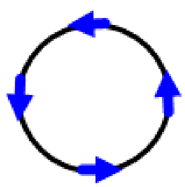
Example 1: Three electric currents are drawn below. In each case, draw the magnetic field that would be created by that electric current. Use the right hand rule. If you find that you fingers have to go into the page on one side of the wire, then the field should go into the page on that side (x's), and out of the page on the opposite side of the wire (dot's)



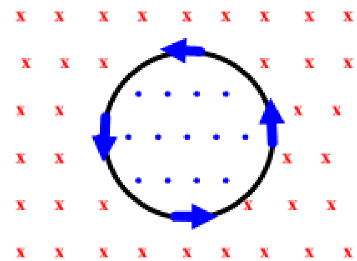
Solution: 

Example 2: A loop of wire has a current running through it. The direction of the current is indicated by the arrows. Draw the induced magnetic field both inside and outside the loop.

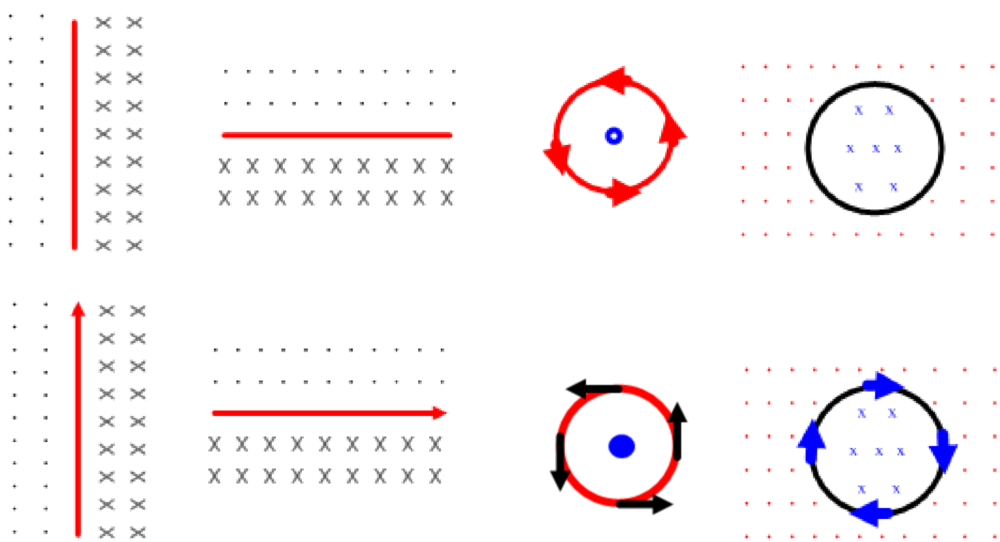
Problem



Solution



Example 3: In each case below, a magnetic field is induced by an electric currents passing through a wire (shown as a solid line or a single dot). In each case, use the given magnetic field and the right hand rule to determine the direction of the current.



Solution

Units and Symbols for Magnetic Field

The symbol for the magnetic field is "B"; sometimes magnetic fields are called "B-fields." There are two common units for measuring B-fields: the Tesla (T) is equal to 1 Newton/Ampere-meter. This naturally flows from the units we used in earlier chapters (Newtons, Amperes and meters) and emerges from the generation of B-fields by electric current, which we'll be studying next.

$$\mathbf{1\ Tesla = 1\,\frac{Newton}{Amp - meter}} \qquad\qquad \mathbf{1\ Gauss = 10^{-4} Tesla}$$

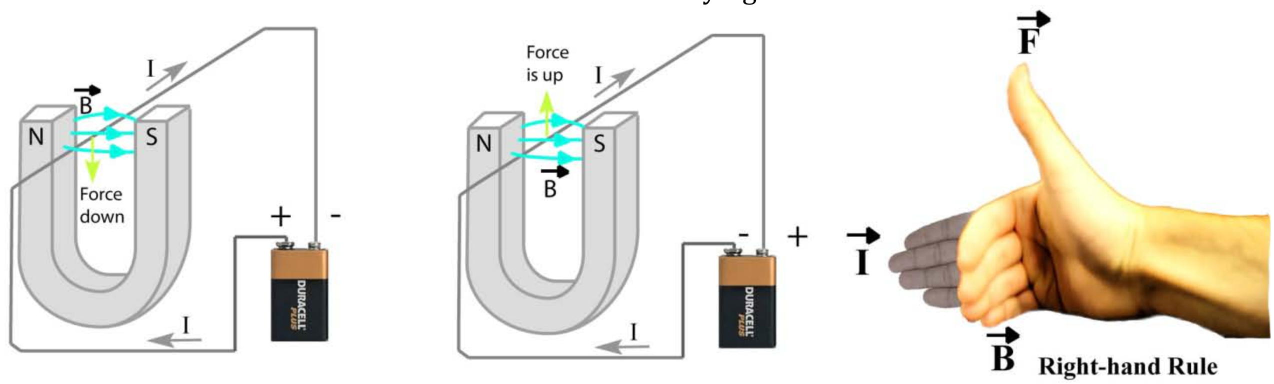
The Gauss (G) is equal to 10^{-4} Tesla (T); it is often used because magnetic fields are usually much less than a Tesla, it's more convenient to measure them in Gauss:

Force on a current carrying wire in a B-field

The connection between electricity and magnetism became clearer when it was observed that a wire carrying an electric current through a magnetic field experienced a force. This magnetic force is due to the interaction between the fixed external magnetic field and the magnetic field created by the electric current. The current running through the wire creates a magnetic field, as was discussed above, and the magnetic field of the wire interacts with the external magnetic field. Just as was the case with two magnets, the fields interact resulting in a force on the sources of both fields, the wire and the fixed magnet. The result is a force both on the wire and on the fixed magnet. However, since the wire has so much less mass than the fixed magnet, the same force acting on both results in the wire's movement being much larger, and easier to detect.

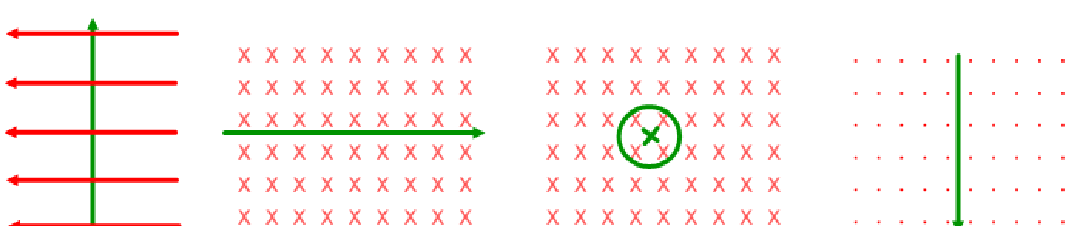
The diagram below illustrates the direction of the force created. This is a truly 3-D situation as the force, current and B-field are all perpendicular to one another (**there is no force if the B-field and electric current are parallel**). A second right hand rule allows us to determine the direction of the force given directions of the field and current.

To determine the direction of the force on a current carrying wire:



1. Point your right arm and hand in the direction that the electric current is flowing.
2. Bend your fingers in the direction that the external magnetic field is going (rotate but don't change the direction your arm is pointing).
3. Extend your thumb so it is perpendicular to your arm and hand. Your thumb is pointing in the direction of the force on the wire.

Example 3: Determine the direction of the force on the wire in each case.



Solution

Note that if the B-field and the current are parallel, there is no force.

The magnitude of the force on the current carrying wire is proportional to the size of the B-field, the size of the electric current and length of the wire. The force is simply equal to the product of these three factors, as long as the field and current are perpendicular. The force is zero if they are parallel. In your next course you will add a factor to account for angles between 0° and 90°, but for now we'll only consider cases where the field and current are either parallel (no force) or perpendicular, in which case the force is given by:

$$F_B = I\,L\,B_{perpendicular}$$

I is the magnitude of the electric current, L is the length of the wire and $B_{perpendicular}$ is the magnetic field perpendicular to the wire. The current is measured in Amperes; the length of wire is measured in meters; the B-field is measured in Teslas; and the force is measured in Newtons.

Example 4: Determine the magnetic force acting on a 2.0-m long wire which is carrying a 4.0-A current perpendicular to a 0.25-T magnetic field.

Solution: $F_B = ILB_{perpendicular}$
$F_B = (4.0A)(2.0m)(0.25T)$
$F_B = 2.0\ N$

Example 5: A 3.0-m long wire experiences a 9.0N force while carrying a 6.0-A current perpendicular to a magnetic field. What is the size of the magnetic field?

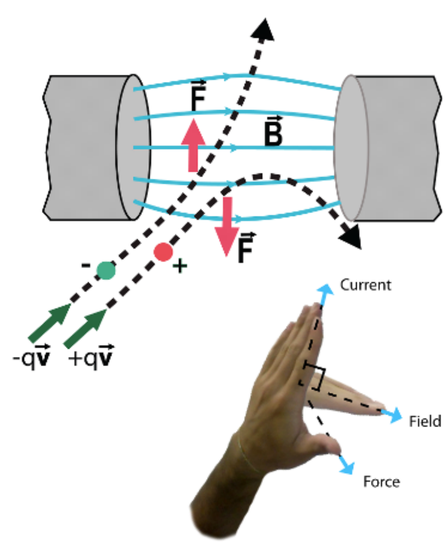
Solution: $F_B = ILB_{perpendicular}$
$B_{perpendicular} = F_B / (IL)$
$B_{perpendicular} = 9.0\text{-N} / (6.0\text{-A})(3.0m)$
$B_{perpendicular} = 0.50\text{-T}$

Force on a moving electric charge in a B-field

Since an electric current is just a moving stream of charges, it is not surprising that a moving charge in a magnetic field experiences a magnetic force which is like that experienced by a current carrying wire. In fact, we'll show below that this is actually the identical force, in a slightly different guise: the force on a current carrying wire is just the result of a force on each of the moving charges within the wire.

The force experienced by a moving charge depends on its velocity, and charge, and the amount of magnetic field. It also depends on the angle of the velocity to the B-field. The force is maximum when the velocity of the charge and the field are perpendicular; zero when they are parallel; and some intermediate value at angles between 0 and 90 degrees. In your next course you will add a factor to account for angles between 0˚ and 90˚, but for now we'll only consider cases where the velocity and B-field are either parallel (no force) or perpendicular, in which case the force is given by:

$$F_B = qvB_{perpendicular}$$

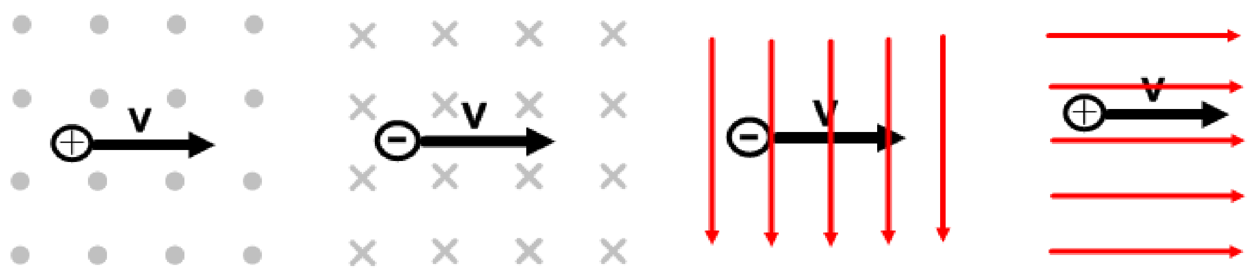


The magnetic force is F_B, the quantity of charge is given by q, the velocity is v and the perpendicular component of external magnetic field is given by $B_{perpendicular}$.

For a positive charge, the direction of the force is given by the same right hand rule that we used to determine the force on a current carrying wire. Just point your right arm and hand in the direction of the velocity, bend your fingers in the direction of the field, and your thumb will point up in the direction of the force; this is identical to what you did to find the force on a current carrying wire. The only difference is that you are now pointing your arm in the direction of the velocity instead of the direction of the current.

However, charges can be positive or negative; if the charge is negative, the force will be in the opposite direction of that found for a positive charge. One way to determine the force on a negative charge is to use the same right hand rule, and then reverse your answer as your final step. If the right hand rule says that the force will be up for a positive charge, then it will be down for a negative charge. Alternatively, you could use your left hand for negative charges; you'll get the same answer as if you used your right hand and then flipped your final answer. Some people prefer to use their left hand since they won't forget to flip the answer at the end. Others prefer to always use one hand; it's up to you.

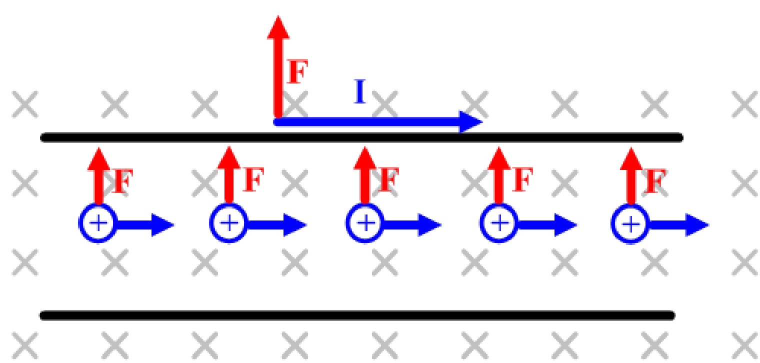
Example 4: Determine the direction of the force on the charge in each case.



Solution:

Moving Charges and Electric Current

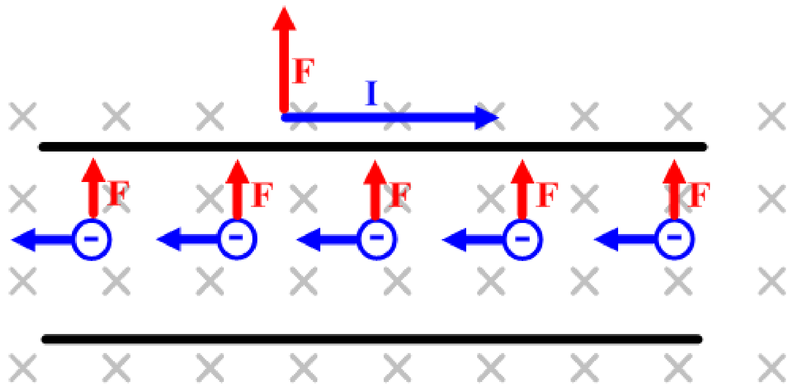
The deep connection between the magnetic force acting on a current in a magnetic field and the magnetic force acting on a moving charge in a magnetic field is made clear by the next two diagrams. In the first, imagine that a current is made up of a large number of positive charges (only a few are shown) flowing through a wire in a magnetic field. The electric current is to the right, in the same direction as the velocity of the charges. The force on the wire is given by the right hand rule for a current carrying wire in a B-field. With the B-field directed into the page and the current to the right, the force will be towards the top of the page. But the source of that force on the wire is the



sum of the force on the all the individual charges that comprise the current in the wire. To find those forces, we use the right hand rule for the force on a moving charge in a B-field. With the field into the page and the velocity to the right, the force is also towards the top of the page. Both these forces (on a current or a charge) are really the same.

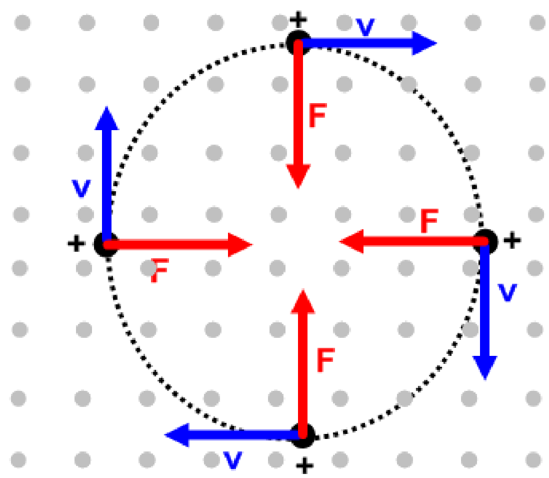
But electric current is actually comprised of negative charges (electrons) moving in a direction opposite to that of the conventional current, the positive charges (protons) are locked in place and unable to move.

However, the result is the same. The direction of the conventional current is still to the right regardless of whether it was due to positive charges moving to the right or negative charges to the left. So the force on the wire is the same, towards the top of the page. The force on the electrons is also towards the top of the page since the right hand rule for a field into the page and velocity to the left would be towards the bottom of the page, but that has to be reversed for negative charges (or if you used the left hand rule for negative charges). Either way, the connection between the force on moving charges and current carrying wires can be seen to have the same source.

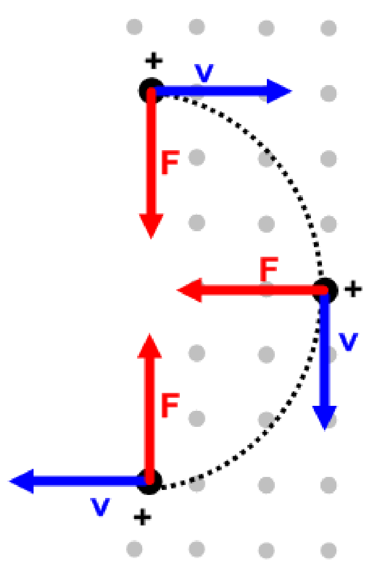


Circular Motion of Charges in a B-Field

The force acting on a charge which is moving perpendicularly to a magnetic field is always perpendicular to the direction of motion: F_B is always perpendicular to v. In an earlier chapter on circular motion, we learned that if the force acting on an object is always perpendicular to its motion that the result will be circular motion, and that the acceleration of the object will be given by $a = v^2/r$. This will be the case for any charge which is traveling through space when a perpendicular magnetic field is suddenly switched on, as shown to the right.



If a moving charge enters a region of space in which a magnetic field was always present, the charge will complete a half circle and then exit the field traveling in a direction opposite to its initial velocity. Its motion will be circular while in the field, but upon completing a half circle, it will exit the field, thereafter it will travel in a straight line, since no force will be acting on it upon exiting the field.



We can use combine the relationships we developed for circular motion in an earlier chapter and what we have learned about the magnetic force acting on a moving charge in this chapter to learn characteristics of the charge by observing its motion in a field. This is exactly what is done in observing nuclear reactions in bubble chambers or in mass spectrometers to study unknown charges.

$\Sigma F = ma$

$F_B = ma_c$ The magnetic force is responsible for the circular motion.

$qvB = \frac{mv^2}{r}$ $F_B = qvB$ and $a_c = \frac{v^2}{r}$

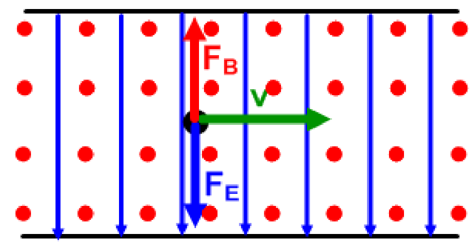
$qB = \frac{mv}{r}$ Divide both sides by v.

$\frac{q}{m} = \frac{v}{Br}$ Solve for the charge to mass ratio.

By measuring the radius of circular motion described by a particle of known velocity in a B-field of known magnitude, we can determine q/m: the charge to mass ratio of the particle. This is very valuable in identifying the identity of an unknown particle. It's the bases of mass spectrometry, which allows unknown samples to be analyzed.

Velocity Selectors

In order to use the above relationship to determine the charge to mass ratio of a particle, we need to know the velocity of the particle. It turns out that it's possible to create a device that allows only particles at a particular velocity to pass through it. By using that velocity selector as the first stage of a mass spectrometer, the velocity of all the particles entering the device will be known.



A velocity selector consists of an Electric Field and a Magnetic Field which are perpendicular to one another, and perpendicular to the velocity of the particles passing through it. The force acting on the particle due to the electric field is given by $F_E = qE$ while the force due to the magnetic field is given by $F_B = qvB$. If the two forces are equal and opposite, the particle will travel straight through. However, this will only be the case for one particular velocity.

$\Sigma F = ma$

$F_B - F_E = 0$ There is no acceleration, so the magnetic force is equal to the electric force.

$F_B = F_E$

$qvB = qE$ $F_B = qvB$ and $F_E = qE$

$vB = E$

$v = \frac{E}{B}$ Solve for v.

If a particle has a lower velocity than E/B, the electric force will be stronger and force the particle to curve downwards. If a particle has a high velocity than E/B, the magnetic force will be larger and the particle will curve upwards. Only in the case where the velocity is given by v=E/B, will a charged particle travel straight through the detector.

Note, that while our drawing is based on a positive particle, it makes no difference whether the particle is positive or negative, since q canceled out in our equations. Similarly, the size of the charge makes no difference; particles will be selected to pass through the detector solely based on their velocity. By measuring the size the electric and magnetic fields, the velocity of all charged particles can be determined. Also, by adjusting either E, B, or both, different velocity particles can be selected.

The size of the Magnetic Field created by a Current Carrying Wire

Above, we determined the direction of the magnetic field created by a current carrying wire: it forms concentric circles around the wire with a direction given by the first Right Hand Rule. The magnitude of the field is given by this expression:

$$\boldsymbol{B = \frac{\mu_0}{2\pi}\frac{I}{r}}$$

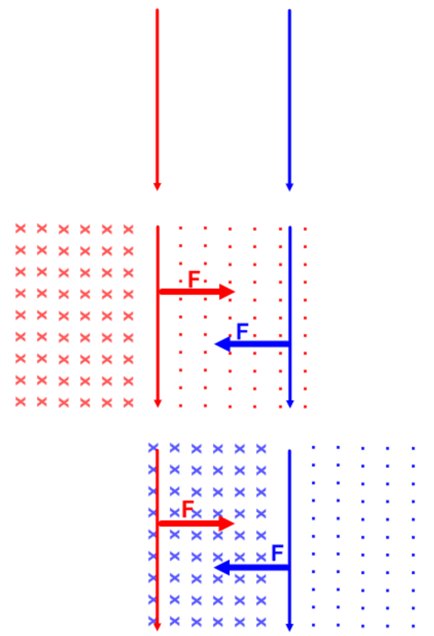
I is the current in the wire and r is the distance from the wire (the field falls off as 1/r). The constant μ_0 is called the permeability of free space and is exactly given by:

$$\mu_0 = 4\pi \times 10^{-7} T \cdot \frac{m}{A}$$

The reason that this is exactly the case is that an Ampere of current is actually defined by the above formula and that specific value of μ_0. When working with current carrying wires this results in a great simplification since the 4π in the definition of μ_0 and the 2π in the formula for B will always cancel leaving only a factor of 2.

Example 6: What is the magnitude of the magnetic field 6.0-m from a wire carrying a 4.0-A current?

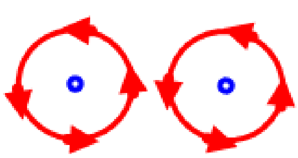
Solution:
$$B = \frac{\mu_0 I}{2\pi r}$$
$$B = \frac{\left(4\pi\times10^{-7}\frac{T\cdot m}{A}\right)(4.0A)}{(2\pi)(6.0m)}$$
$$B = 1.3 \times 10^{-7} T$$



The Magnetic Force Between Two Current Carrying Wires

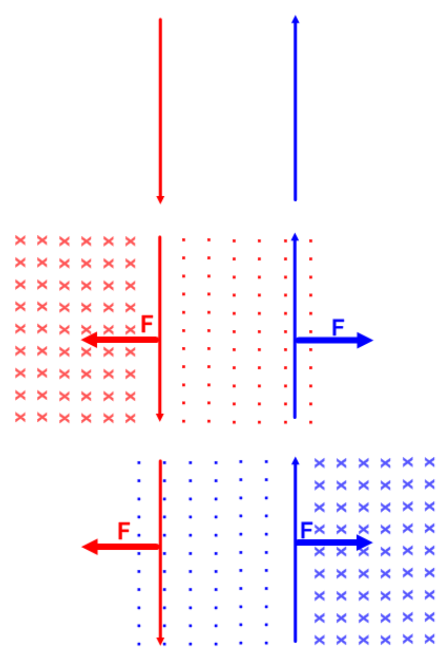
Since a current carrying wire creates a magnetic field, and a current carrying wire in a magnetic field experiences a magnetic force, then two current carrying wires must create fields that interact and create a force between them.

In the first diagram, we are looking down on two wires carrying current in the same direction. In the second illustration, we see the magnetic field created by the wire on the left (using our first right hand rule), and the force it creates on the wire on the right (using our second right hand rule). The force on the left wire is then due to Newton's Third Law, or alternatively, we can see, in the third diagram, that the force on the leftmost wire must be the same by going through the same steps that we did above, but switching the roles of the wires. We have the wire on the right create the field and the wire on the left react to it. We get the same result: **Wires carrying currents in the same direction attract.**

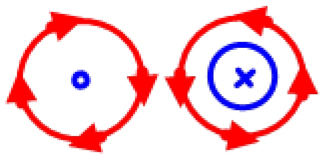


Another way to think about this is to look at the wires from the perspective that the currents are running towards you, instead of looking at the wires from above.

In this diagram two wires are carrying current towards you. Notice that in the region between the two wires that their fields oppose one another, reducing the net field: the field of the left wire is directed up while that of the wire on the right is directed down. This result in a force pushing the wires together since there is less energy stored in a smaller net field, and systems move in ways to minimize their energy. This gives us the same result as above, but from a different perspective.



If the currents run in opposite directions, the wires will repel. This is shown below by reversing the direction of the current in the wire on the right and following the same set of steps we did before. Pick a wire and draw the field it creates using the first right hand rule. Then determine the force on the right wire using the second right hand rule. Lastly, we show that we could have picked the other wire to generate the field and we'd get the same result: **Wires carrying currents in opposite directions repel.**



On the other hand, if the wires are carrying current in opposite directions, their fields will be aligned in the space between them. This increases the net field, and represents a higher energy state, so the wires will feel a force to decrease that interaction: the wires will repel one another so that they will move farther apart, decreasing the strength of the field that each will experience due to the other.

We can determine the magnitude of the force between two wires by assuming that one wire creates a magnetic field and the second wire responds to it. Because of Newton's Third Law, each wire will feel an equal and opposite force, so which wire you choose as the one to create the field doesn't matter, you'll get the same result.

The force on a current carrying wire due to an external field is given by F = BIL. Let's call the current in the wire that will respond to the external field I_1. Then this become $F_1 = BI_1L$. The external field is created by the second current carrying wire. The magnitude of the field it creates is given by: $B = \frac{\mu_0 I}{2\pi r}$, labeling the current in this wire I_2. Let's then substitute this for B in the first expression.

$$F_1 = BI_1L$$

$$F_1 = \left(\frac{\mu_0 I_2}{2\pi r}\right) I_1 L$$

$$F_1 = \frac{\mu_0 I_2 I_1 L}{2\pi r}$$

$$\frac{F}{L} = \frac{\mu_0 I_2 I_1}{2\pi r}$$

The force per unit length, F/L, is given by this expression. The force will be equal and opposite on each wire, so we can replace F_1 by F. Only the product of the currents appears, so it's clear that the force on each wire must be the same. We assume that the wires have equal lengths.