

8th Grade Mathematics

Unit 1 Curriculum Map: September 9th – October 25th



ORANGE PUBLIC SCHOOLS
OFFICE OF CURRICULUM AND INSTRUCTION
OFFICE OF MATHEMATICS

Common Core Standards

| REVIEW OF GRADE 7 FLUENCIES | |
|-----------------------------|---|
| <u>7.NS.2d</u> | Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. |
| <u>7.NS.3</u> | Solve real-world and mathematical problems involving the four operations with rational numbers |

| GRADE 8 CONTENT | |
|-----------------|---|
| <u>8.NS.1</u> | Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational. |
| <u>8.NS.2</u> | Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. |
| <u>8.EE.2</u> | Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. |
| <u>8.G.6</u> | Explain a proof of the Pythagorean theorem and its converse |
| <u>8.G.7</u> | Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. |
| <u>8.G.8</u> | Apply the Pythagorean theorem to find the distance between two points in a coordinate system. |

Model Curriculum Student Learning Objectives

| SLO | Description |
|-----|--|
| 1 | Compare rational and irrational numbers to demonstrate that the decimal expansion of irrational numbers do not repeat; show that every rational number has a decimal expansion which eventually repeats and convert such decimals into rational numbers. |
| 2 | Use rational numbers to approximate and locate irrational numbers on a number line and estimate the value of expressions involving irrational numbers. |
| 3 | Evaluate square roots and cubic roots of small perfect squares and cubes respectively and use square and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number. |
| 4 | Identify $\sqrt{2}$ is irrational |
| 5 | Explain a proof of the Pythagorean theorem and its converse |
| 6 | Utilize the Pythagorean theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems |
| 7 | Use the Pythagorean theorem to determine the distance between two points in the coordinate plane. |

Connections to the Mathematical Practices

| | |
|---|--|
| 1 | Make sense of problems and persevere in solving them |
| | <ul style="list-style-type: none"> - Persevere in approximating irrational numbers - Analyze Pythagorean problems to determine the placement of numbers in a right triangle in a real world situation - Look for efficient ways to represent and solve a problem |
| 2 | Reason abstractly and quantitatively |
| | <ul style="list-style-type: none"> - Use concrete numbers to explore the properties of numbers in exponential form - Use symbols to represent integer exponents and make sense of those quantities in problem situation - Contextualize to understand the meaning of a number - Decontextualize to manipulate symbolic representations by applying properties of operations |
| 3 | Construct viable arguments and critique the reasoning of others |
| | <ul style="list-style-type: none"> - Reason through the acceptability of definitions and proofs - Analyze situations and break them into cases - Examine the implications of definitions and proofs on existing properties of integer exponents. - Focus on the logic argument while attending to details that develop during the reasoning process |
| 4 | Model with mathematics |
| | <ul style="list-style-type: none"> - Interpret real world situations in terms of the Pythagorean theorem - Use strategies to approximate an irrational number |
| 5 | Use appropriate tools strategically |
| | <ul style="list-style-type: none"> - Use a coordinate grid to determine the length of a hypotenuse - Approximate irrational numbers on a number line - Draw pictures to represent the relationship of the sides of a right triangle |
| 6 | Attend to precision |
| | <ul style="list-style-type: none"> - Attend to the definitions provided throughout the topics and express them clearly (rational, irrational, integer, hypotenuse) |
| 7 | Look for and make use of structure |
| | <ul style="list-style-type: none"> - Understand and make analogies to the distributive law as they develop properties of exponents - Examine patterns in decimals for repetition |
| 8 | Look for and express regularity in repeated reasoning |
| | <ul style="list-style-type: none"> - While evaluating the cases developed for the proofs, students identify when a statement must be proved or if it has already been proven - Use repeated reasoning to understand algorithms and generalize patterns - Use iterative processes to determine more precise rational approximations for irrational numbers. - Analyze patterns of repeating decimals to identify the corresponding fraction |

Vocabulary

| Term | Definition |
|--|---|
| <i>Additive Inverse</i> | Two numbers whose sum is 0 are additive inverses of one another. Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverse of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$ |
| <i>Algebraic Expression</i> | A mathematical phrase involving at least one variable. Expressions can contain numbers and operation symbols. |
| <i>Addition Property of Equality</i> | For real numbers a, b, and c, if $a = b$, then $a + c = b + c$. In other words, adding the same number to each side of an equation produces an equivalent equation. |
| <i>Cube Root</i> | If the cube root of a number b is a (i.e., $\sqrt[3]{b} = a$), then $a^3 = b$. |
| <i>Decimal Expansion</i> | The decimal expansion of a number is its representation in base 10 (i.e., the decimal system). For example, the decimal expansion of 25^2 is 625, or π is 3.14159..., and of $\frac{1}{9}$ is 0.1111... |
| <i>Equation</i> | A mathematical sentence that contains an equals sign |
| <i>Exponent</i> | The number of times a base is used as a factor of repeated multiplication |
| <i>Inverse Operation</i> | Pairs of operations that undo each other, for example, addition and subtraction are inverse operations and multiplication and division are inverse operations |
| <i>Evaluate an Algebraic Expression</i> | To perform operations to obtain a single number or value |
| <i>Irrational</i> | A real number whose decimal form is non-terminating and non-repeating that cannot be written as the ratio of two integers |
| <i>Multiplication Property of Equality</i> | For real numbers a, b, and c ($c \neq 0$), if $a = b$ and $ac = bc$. In other words, multiplying both sides of an equation by the same number produces an equivalent expression |
| <i>Multiplicative Inverses</i> | Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$. |
| <i>Perfect Square</i> | A number that has a rational number as its square root. |
| <i>Radical</i> | A symbol $\sqrt{\quad}$ that is used to indicate square roots |
| <i>Rational</i> | A number that can be written as the ratio of two integers with a nonzero |

| | |
|--|---|
| | denominator |
| <i>Square Root</i> | One of two equal factors of a nonnegative number. For example, 5 is a square root of 25 because $5 \times 5 = 25$. Another square root of 25 is -5 because $(-5) \times (-5) = 25$. The +5 is called the principle square root of 25 and is always assumed when the radical symbol is used. |
| <i>Altitude of a Triangle</i> | The perpendicular distance between a vertex of a triangle and the side opposite that vertex. Sometimes called the height of a triangle. Also, sometimes the line segment itself is referred to as the altitude. |
| <i>Base</i> | For two-dimensional figures, any side can be a base. Typically, however, the bottom side, on which the polygon 'sits,' it's called the base. |
| <i>Coordinate Plane</i> | A two-dimensional surface on which points are plotted and located by their x and y coordinates. |
| <i>Coordinate Point of a Plane</i> | A pair of numbers defining the position of a point on a two-dimensional plane |
| <i>Converse of the Pythagorean Theorem</i> | If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, the triangle is a right triangle |
| <i>Cubed Root</i> | One of three identical factors of a number that is the product of those factors |
| <i>Deductive Reasoning</i> | The process by which one makes conclusions using known facts, definitions, rules, or properties |
| <i>Hypotenuse</i> | The hypotenuse is the side of a right triangle that is directly across from the right angle |
| <i>Leg of a Triangle</i> | Either of the two shorter sides of a right triangle. These two sides together form the right angle in the right triangle |
| <i>Literal Equation</i> | An equation with more than one variable. Formulas are literal equations. |
| <i>Perfect Squares</i> | The product of a rational number multiplied by itself |
| <i>Perfect Cubes</i> | In arithmetic and algebra, the cube of a number n is its third power. It is the result of the number multiplying by itself three times. |
| <i>Pythagorean Theorem</i> | The theorem states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs. |
| <i>Pythagorean Triples</i> | A set of positive integers a, b, and c that fit the rule $a^2 + b^2 = c^2$ |
| <i>Right Triangle</i> | A triangle with exactly one right angle |

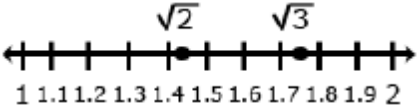
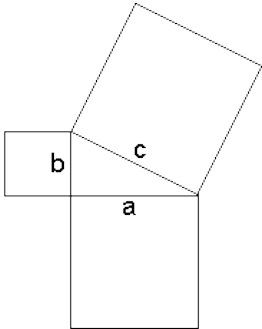
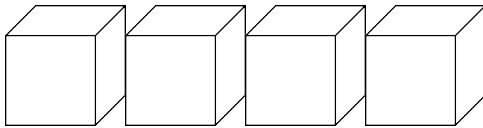
Potential Student Misconceptions

- Students fail to realize a ratio can be a fraction and all fractions are ratios: Explain to students that both ratios and fractions compare two quantities. However a ratio can express a part-to-whole comparison or a part-to-part comparison, where as a fraction is always a part-to-whole comparison.
- Students believe repeating decimals are irrational: Show the student examples of fractions that are repeating decimals, such as $\frac{1}{3}$. Because the repeating decimal can be written as a fraction, by definition, makes it a rational number.
- Students commonly believe that decimals with a pattern are rational. For example, students often think $0.010010001\dots$ is rational. This decimal is great example to show students. They know what is coming next; however it does not repeat and therefore is irrational.
- Students often misunderstand the relationship between rational numbers and its subgroups. Show examples of natural numbers, whole numbers, and integers and how they relate to each other. The graphic organizer of nested subsets or a foldable is a powerful way to show the relationships.
- Students do not see the fraction bar as an operation: A fraction is a division problem and a division problem can be represented with a fraction.
- Students do not see irrational numbers as numbers that do not repeat or terminate. Some students are surprised that the decimal representation of Pi does not repeat. Some students believe that if only we keep looking at digits farther to the right, eventually a pattern will emerge. A few irrational numbers are given special names (Pi and e), and much attention is given to square root of 2. Because we name so few irrational numbers, students sometimes conclude that irrational numbers are unusual and rare. In fact, irrational numbers are much more plentiful than rational numbers in the sense that they are “denser” in the real number system. Using square roots of non-perfect squares is a powerful way to explain this.
- Students commonly think that an exponent means to multiply the base times the exponent. For example, students think that $3^2 = 3 \times 2$ as opposed to 3×3 or 2^3 to be 2×3 instead of $(2)(2)(2)$. Students need to understand the fact that, x^3 , means $x \times x \times x$, means x times x, times x, not $3x$ or 3 times x.
- Students mistakenly apply the sign of the exponent to the base, that is students think $x^{-n} = -x^n$. For example, students believe 3^{-2} to be the same as -3^2 or -9 instead of $\frac{1}{3^2}$. Patterns are a great way to show the progression from positive exponents to negative exponents.
- Students do not understand that the square root problems have two possible answers and this should be denoted using a \pm symbol. For example, if students are asked to find the square root of 25, $\sqrt{25}$, there are two possible answers, 5 or -5. So when solving a problem such as $x^2 = 25$, students should take the square root of both sides leaving $x =$

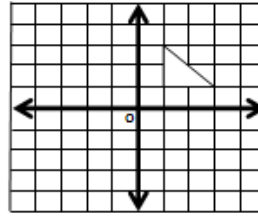
$\pm \sqrt{25}$. By using the \pm symbol, they are showing that they understand that there are two possible answers, one positive and one negative.

- Students do not identify the variables as different legs on a right triangle: There is no connection made between the formula and the definition. Rather than saying $a^2 + b^2 = c^2$, it is more useful for them to say $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$.
- Students commonly do not understand that the square terms are the squares drawn off of each side of the right triangle. For example, $a^2 + b^2 = c^2$ is in reference to the three squares that you draw off the legs and hypotenuse of the right triangle. It is not the sum of the lengths of the legs that will equal the length of the hypotenuse; it is the area of the squares off the legs that will sum to equal the area of the square off the hypotenuse.
- Students believe that the Pythagorean Theorem applies to all triangles. It is important to emphasize that Pythagorean Theorem includes the conditional statement that if you have a right triangle, then $a^2 + b^2 = c^2$. Students need opportunities to try Pythagorean theorem on all different types of triangles to determine which types of triangle the theorem holds true for.

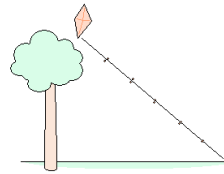
Teaching Multiple Representations

| CONCRETE REPRESENTATIONS | |
|---|--|
| <ul style="list-style-type: none"> • Number Lines • Pattern Blocks • Unit Cubes |    |
| PICTORIAL REPRESENTATIONS | |
| <ul style="list-style-type: none"> • Graphic Organizer | <p style="text-align: center;">Real Numbers</p> <p style="text-align: center;">All real numbers are either rational or irrational</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px; text-align: center;"> Rational <div style="border: 1px solid black; padding: 2px; margin: 2px; text-align: center;">Integers</div> <div style="border: 1px solid black; padding: 2px; margin: 2px; text-align: center;">Whole</div> <div style="border: 1px solid black; padding: 2px; margin: 2px; text-align: center;">Natural</div> </div> <div style="border: 1px solid black; padding: 5px; text-align: center;">Irrational</div> </div> |

- **Coordinate Grid**



- **Picture Representation**



ABSTRACT REPRESENTATIONS

- The Pythagorean Theorem: $a^2 + b^2 = c^2$
- Use of square and cube root symbols

Pacing Guide

| Activity | Common Core Standards/SLO | Estimated Time |
|---|---|--------------------------|
| Review of 7 th Grade | 7.NS.2d; 7.NS.3 | 8 days |
| Looking for Pythagoras (CMP3) Investigation 1 | 8.G.6 SLO 5, 7 | 3 days |
| Looking for Pythagoras (CMP3) Investigation 2 | 8.EE.2; 8.NS.2; 8.G.8; 8.G.6; 8.G.7 SLO 2, 3, 5, 6, 7 | 4 days |
| Looking for Pythagoras (CMP3) Investigation 3 | 8.G.6; 8.G.7; 8.G.8 SLO 5, 6, 7 | 4 days |
| Assessment Check 1 | 8.NS.2; 8.EE.2; 8.G.6; 8.G.7; 8.G.8 | 1 day |
| Looking for Pythagoras (CMP3) Investigation 4 | 8.EE.2; 8.G.7; 8.NS.1; 8.NS.2 SLO 1, 2, 3, 4, 6 | 3 days |
| Looking for Pythagoras (CMP3) Investigation 5 | 8.G.7; 8.G.8; 8.G.4; 8.NS.1 SLO 6, 7 | 3 days |
| Assessment Check 2 | 8.NS.1; 8.G.6; 8.G.7; 8.G.8 | 1 day |
| Unit Review/Task/Project | 8.NS.1; 8.NS.2; 8.EE.2; 8.G.6; 8.G.7; 8.G.8 | 5 days |
| Unit 1 Assessment | 8.NS.1; 8.NS.2; 8.EE.2; 8.G.6; 8.G.7; 8.G.8 | 1-2 days (October 24/25) |

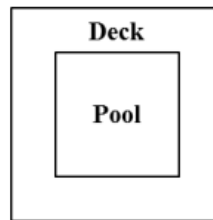
Assessment Checks

Assessment Check 1

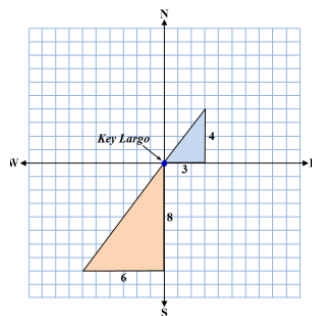
1. Moesha has 196 pepper plants that she wants to form in square formation. How many pepper plants should she plant in each row?
2. Graph the numbers $\sqrt{8}$, 1.4, $\sqrt{64}$, $\frac{1}{2}$, π on a number line. Then, order the numbers from least to greatest.



3. Randall's dad is installing a new pool in their backyard. The pool is a square and has an area of 121 ft². Randall's dad will then build a 4 ft wide deck to surround the pool. What is the outside perimeter of the deck?



4. What kind of triangle is it when $a^2 + b^2 < c^2$?
 A. Right Triangle B. Obtuse Triangle C. Acute Triangle
5. Two sailboats leave Key Largo, Florida. One of the boats travels 3 miles east then 4 miles north. The second sailboat travels 8 miles south and 6 miles west. How far apart, in miles, are the boats?



Assessment Check 2

1. A student made this conjecture and found two examples to support the conjecture.

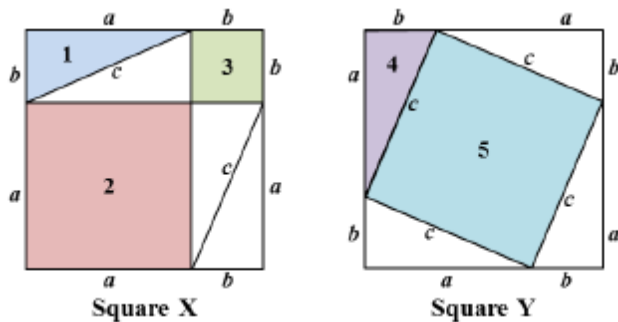
“If a rational number is not an integer, then the square root of the rational number is irrational. For example, $\sqrt{3.6}$ is irrational and $\sqrt{\frac{1}{2}}$ is irrational.”

Provide two examples of non-integer rational numbers that show that the conjecture is false.

Example 1:

Example 2:

2. Use the two diagrams to answer the question.



The diagram shows two congruent squares divided in two different ways. For both squares, each side has been divided into the same two lengths, a and b .

Part A

Give the area of each shaded region in terms of a , b , and/or c .

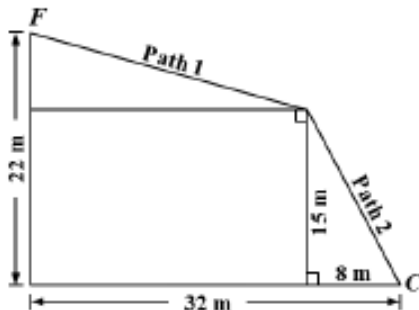
| | |
|----------|--|
| Area 1 = | |
| Area 2 = | |
| Area 3 = | |
| Area 4 = | |
| Area 5 = | |

Part B. The diagram provides a basis for proving the Pythagorean Theorem for right triangles, $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of the right triangle and c is the length of the hypotenuse.

The statements in the table below prove that $a^2 + b^2 = c^2$. For each statement write a brief justification of the truth of the statement in the right-hand column.

| Step | Statement | Justification |
|------|--|-------------------------------|
| 1 | Each side of each square is divided into the same two parts, a and b | Given |
| 2 | Squares X and Y are congruent | Given |
| 3 | Area of Square X = Square Y | Squares X and Y are congruent |
| 4 | Area of Square X = $a^2 + b^2 + 4\left(\frac{ab}{2}\right) = a^2 + b^2 + 2ab$ | |
| 5 | Area of Square Y = $c^2 + 4\left(\frac{ab}{2}\right) = c^2 + 2ab$ | |
| 6 | $a^2 + b^2 + 2ab = c^2 + 2ab$ | |
| 7 | $a^2 + b^2 = c^2$ | |

3. Fernando followed two diagonal paths, Path 1 and 2, to get from his house, F, to a neighborhood corner store, C, as shown below. What is the total distance of the two paths, in meters (m), from F to C?



Extensions

Online Resources

<http://www.illustrativemathematics.org/standards/k8>

- Performance tasks, scoring guides

<http://www.ixl.com/math/grade-8>

- Interactive, visually appealing fluency practice site that is objective descriptive

https://www.khanacademy.org/math/geometry/right_triangles_topic/pyth_theor/v/pythagorean-theorem

- Tracks student progress, video learning, interactive practice

<https://www.khanacademy.org/math/arithmetic/order-of-operations/rational-irrational-numbers/e/recognizing-rational-and-irrational-numbers>

- Tracks student progress, video learning, interactive practice

http://www.doe.k12.de.us/assessment/files/Math_Grade_8.pdf

- Common Core aligned assessment questions, including Next Generation Assessment Prototypes

Assessment Resources
8.NS Summative Task

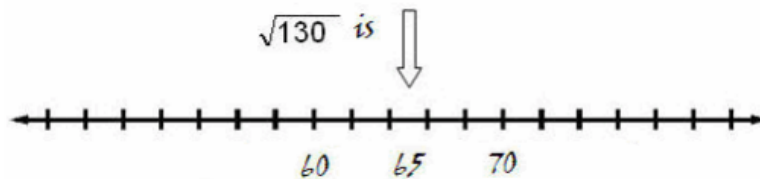
1. Analyze Robin's reasoning in her answer to the test question about rational or irrational numbers. Does she have a strong understanding of rational and irrational numbers? Does her reasoning make sense? If not, what misconceptions does she have about this topic? How would you describe

3rd Block

Robin Radical

1. Is $\sqrt{130}$ rational or irrational? Where would $\sqrt{130}$ be located on a number line? Explain your reasoning.

$\sqrt{130}$ is a rational number because 130 is even. All rational numbers are even and irrational numbers are odd when the numbers are under the square root sign. The square root sign is the opposite of squaring a number. Squaring a number is the same as raising it to the 2nd power. So, to find the value of a number under the square root sign, you divide it by 2. So, $130 \div 2$ is 65. The answer is 65.



8.EE.2 Summative Task

Classify the numbers as perfect squares and perfect cubes. To classify a number, place it into the appropriate column in the chart. Numbers that are neither perfect squares nor perfect cubes should not be placed in the chart.

| | | | | | | | |
|---|----|----|-----|-----|-----|-----|-----|
| 1 | 64 | 96 | 125 | 200 | 256 | 333 | 361 |
|---|----|----|-----|-----|-----|-----|-----|

| Perfect Squares but NOT Perfect Cubes | Both Perfect Squares and Perfect Cubes | Perfect Cubes but NOT Perfect Squares |
|---------------------------------------|--|---------------------------------------|
| | | |

8.G.6-8 Summative Task

Juice Box Problem



A juice box has a base of 6 cm by 8 cm and a height of 12 cm. A straw is inserted into a hole in the center of the top. The straw must stick out 2 cm so you can drink from it.

- If the straw must be long enough to touch each bottom corner of the box, what is the minimum length the straw must be? (Assume the diameter of the straw is 0 for the mathematical model.) You must show all your work and state a clear explanation.
- Include a sketch of the juice box labeling the dimensions.