5th Grade Mathematics

Fractions- Unit 3 Curriculum Map January 6th – March 7th

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Table of Contents

I.	Unit Overview	p. 3
II.	Important Dates and Calendar	p. 4
III.	Curriculum Guide	p. 5
IV.	Review Content Overview	p. 6
V.	Structure of the Modules	p. 8
VI.	Common Core Standards – Review Content	p. 9 - 28
VII.	Assessment Check for Review Content	p. 29 - 35
VIII.	Common Core Standards – Fractions	p. 37
IX.	Connections to Mathematical Practices	p. 39
X.	Vocabulary	p. 40
XI.	Potential Misconceptions	p. 43
XII.	New Content Resources	p. 44 - 61
XIII.	Assessment Check 2	p. 62 - 79
XIV.	Extensions and Sources	p. 80

Common Core Grade Fluency

REVIEW OF GRADE 4 FLUENCY	
4.NBT.4	Fluently add and subtract multi digit whole numbers using the standard algorithm.

EXPECTED 5TH GRADE FLUENCY	
5.NBT.5	Fluently multiply multi-digit whole numbers using standard algorithm.

Unit Overview

<u>In this unit, students will ….</u>

- Use multiple strategies to find equivalent fractions
- Find and generate equivalent fractions and use them to solve problems
- Simplify fractions
- Use concrete, pictorial, and computational models to find common denominators
- Use fractions (proper and improper) and add and subtract fractions and mixed numbers with unlike denominators to solve problems
- Use concrete, pictorial, and computational models to multiply fractions
- Use concrete, pictorial, and computational models to divide unit fractions by whole number and whole numbers by unit fractions
- Estimate products and quotients

- # Important Dates and Calendar

<table>
<tr><th>Week of …</th><th>Monday</th><th>Tuesday</th><th>Wednesday</th><th>Thursday</th><th>Friday</th></tr>
<tr><td>1/6-1/10</td><td colspan="5" rowspan="2">REVIEW MODULES</td></tr>
<tr><td>1/13-1/17</td></tr>
<tr><td>1/21-1/24</td><td>No School</td><td colspan="3"></td><td>Checkpoint</td></tr>
<tr><td>1/27-1/31</td><td colspan="4" rowspan="3">UNIT 3 NEW CONTENT</td><td>1/2 Day</td></tr>
<tr><td>2/3-2/7</td><td rowspan="2"></td></tr>
<tr><td>2/10-2/14</td></tr>
<tr><td>2/17-2/21</td><td colspan="5">NO SCHOOL</td></tr>
<tr><td>2/24-2/28</td><td colspan="5">UNIT 3 NEW CONTENT</td></tr>
<tr><td>3/34-3/7</td><td colspan="5">Assessment Week</td></tr>
</table>

IMPORTANT DATES

Monday, Jan 20th	MLK Day
Friday, Jan 24th	Checkpoint 2 Grades 6-7
Friday, Jan 31st	1/2 Day
Week of Feb 17th	VACATION
Friday, March 14th	Data Due
Friday, March 21st	Data Returned to Principals

5th Grade

Review Content

Activity	Teach Notes	Common Core Standards/SLO	Estimated Time
Module	Review	5.NBT.1 - 2	5 - 10 days
Practice Problems	Review	5.NBT.5 – 6	2 days
Practice Problems	Review	5.NF.3	2 days
Assessment Check (Review)		*See above*	1 day

New Content

Checkpoint #2 (Friday, January 24th)		SGO Standards	1 day
5th Grade Fraction Module 5. NF. 1 - 2	Lessons 1 - 3	5.NF.1 & 2	7 days
5th Grade Fraction Module5. NF. 1 - 2	Lesson 4: Golden Problem	5.NF.1 & 2	1 day
5th Grade Fraction Module 5. NF. 4	Lessons 1 - 4	5.NF.4	7 days
5th Grade Fraction Module 5. NF. 4	Lesson 5: Golden Problem	5.NF.4	1 day
Various Text Resources		5.NF.5	5 days
Unit 3 Assessment		5.NF.1, 5.NF.2, 5.NF.4 a, b, 5.NF.5 a,b,	2 days

Review Content Overview

Weeks One - Three

5.NBT. 1 & 2

Students will develop a mathematical understanding of place value relating to multi-digit whole and decimal numbers. Students will be asked to explain patterns in the number of zeroes of resulting products when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

5.NBT. 5 & 6

Students will be able to determine the size of a product based on the factors (relative to 1), and show division of whole numbers with one and two-digit divisors using place value, arrays, area models, and other strategies.

5. NF. 3

Students will be able to interpret fractions as the division of the numerator by the denominator.

Review

Structure of the Modules

The Modules embody 3 integrated frameworks that promote the development of conceptual and problems solving skills and computational fluency. The **conceptual framework** of the Modules builds from the concrete to the pictorial to the abstract (and the constant blending of each) to help students develop a deeper understanding of mathematics. The Modules also reference a **multiple representations framework** that encourages teachers to present content in multiple modalities to support flexible thinking. These frameworks go beyond concrete representation (i.e. manipulatives) to promote the realistic representation of concepts addressed in multiple settings. Lastly, the Modules embody a '**gradual release' framework** that encourages teachers to progress from whole group to collaborative and finally to an independent practice format.

OVERVIEW
Each module begins with an overview. The overview provides the standards, goals, prerequisites, mathematical practices, and lesson progression.

INTRODUCTORY TASKS
The Introductory Tasks serve as the starting point for the referenced standard and are typically either diagnostic, prerequisite or anticipatory in nature.

GUIDED PRACTICE
Serves for additional teacher guided instruction for students who need the additional help. The tasks can be modeled with students.

COLLABORATIVE PRACTICE
Serve as small group, or partnered work. The work should promote student discourse, which allows students to make sense of problems and persevere in solving them (MP.1). Through teacher-facilitated, whole group discussion, students will have the opportunity to critique the reasoning of others (MP.3).

JOURNAL QUESTIONS
Provide the opportunity to individual, independent reflection and practice. This independent format encourages students to construct viable arguments (MP.3) and to reason abstractly/quantitatively (MP.2).

HOMEWORK
Can be used as additional in-class practice, Independent Practice, etc. This work should be reviewed and discussed. Procedural fluencies are reinforced within this section.

GOLDEN PROBLEM
The Golden Problem is a performance task that reflects an amalgamation of the skills addressed within the Module. The Golden Problem assesses the student's ability to apply the skills learned in a new and non-routine context. More than one-step; problems usually require intermediate values before arriving at a solution (contextual applications). In the US, we see one step problems that require either recall or routine application of an algorithm.

Common Core Standards: 5.NBT.1 & 2

Understand the place value system.

Goals:

Within this module, students will develop a mathematical understanding of place value relating to multi-digit whole and decimal numbers. Students will be asked to explain patterns in the number of zeroes of resulting products when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

Prerequisite Skills:

- Understand place value for multi-digit whole numbers
- Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form
- Compare multi-digit numbers using symbols
- Use place value to round whole numbers

Essential Questions:

How does one identify the place value of whole numbers and decimals?

How does a digit's position affect its value?

What different relationships exist between units in the base-ten number system?

Embedded Mathematical Practices

MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure
MP.8 Look for and express regularity in repeated reasoning

LESSON 4
5.NBT.2
Golden Problem

LESSON 3
5.NBT.1
Golden Problem

LESSON 2
5.NBT.2
Explain patterns in the placement of the decimal point when a decimal is divided by a power of 10.

LESSON 1
5.NBT.1/2
Read, write, and identify place value of whole numbers and decimals.
Explain patterns in the number of zeroes of product when multiplying a number by powers of 10.

Lesson Structure
Introductory Task
Guided Practice
Collaborative Work
Journal Questions
Skill Building
Homework

Lesson 1: Introductory Task “Reaching One Million with Powers of Ten”

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

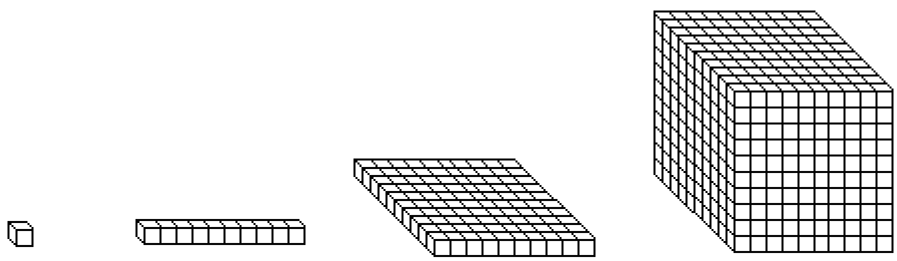
PREREQUISITE COMPENTENCIES FOR THIS TASK

- Understand place value for multi-digit whole numbers
- Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form
- Compare multi-digit numbers using symbols
- Use place value to round whole numbers

Focus Question:

How can numbers be represented?

1. A 1-block is 1 cube. Ten of the 1-blocks have been lined up end to end to make a 10-block. What is the value of a 10-block?



1	10	100	1000
1-block	**10-block**	**100-block**	**1,000-block**

2. 10-blocks can be arranged to form a 100-block. How many 10-blocks are needed to make 100?

3. 100-blocks can be arranged to form a 1000-block. How many 100-blocks are needed to make a 1000-block?

4. What number makes the following equations true?

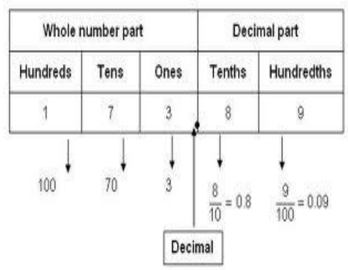
☐x 1=10 **☐x 10=100** **☐x 100=1000**

Multiple Representations Framework

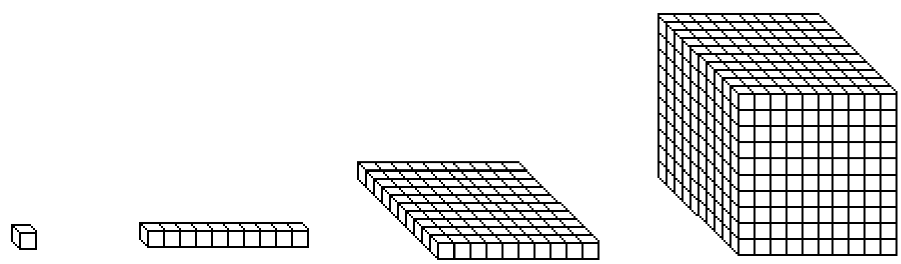
Concrete and Pictorial Representations

Place Value Chart

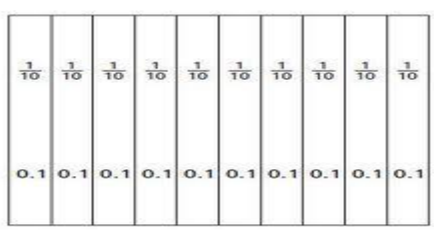
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths



Base Ten Blocks



Fractional Strips



Abstract Representations

Powers of Tens

$100 = 10 \times 10 = 10^2$

Lesson 1 – Guided Practice "Reaching One Million with Powers of Ten"

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

1. How many 1-blocks are needed to make 10? What are the dimensions?
   (Teacher note: Have students arrange the 1-blocks side by side to form the 10-block.)

2. How many 10-blocks are needed to make 100? What are its dimensions?
   (Teacher note: Have students arrange the 10-blocks to form a 10 by 10 square.)

3. How many 100-blocks are needed to make 1,000? What are its dimensions?
   (Teacher note: Have students arrange the 100-blocks to form a 10 by 10 by 10 cube.)

4. How many of the 1,000-blocks are needed to make 10,000? What are its dimensions?
   (Teacher note: Have students place ten (10) of the 1,000-blocks side by side.)

5. How many of the 10,000-blocks are needed to make 100,000? What are its dimensions?
   (Teacher note: The students would place one hundred of the 1000-blocks in a 10 (1000-block) by 10 (1000-block) configuration.)

6. How many of the 10,000-blocks are needed to make 1,000,000? What are its dimensions?
   (Teacher note: Students would place one thousand of the 1,000-blocks in a 10 (1000-block) by 10 (1000-block) by 10 (1000-block) configuration.)

Discuss any patterns specifically relating to powers of 10. The discussion will lead into the Collaborative Practice work on page 6.

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

1. Complete the table below using the patterns noted from the Guided Practice.

1. A box of pencils holds 12 pencils. Complete the equation below to represent the number of pens in 10 boxes. Explain how the equation represents multiplying by 10.
   12+12+12+12+12+12+12+12+12+12=_____

2. Complete the equation below to represent the number of pencils in 10 boxes.
   10x12=_____

3. Explain a rule for multiplying by 10; 100; 1000; any whole number power of 10?

4. A student says that she multiplies by 10 by moving the decimal point one place to the right. Explain why she does this.

5. Complete the equation below.

   1,000x ☐ =240,000

Lesson 1 – Homework "Reaching One Million with Powers of Ten"

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

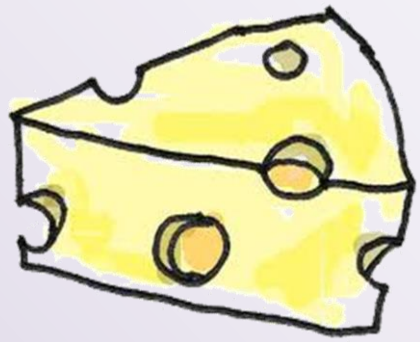
Use the chart to see the value of 1 in different place-value positions.

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
					1
				1	**0**
			1	**0**	**0**
		1	**0**	**0**	**0**
	1	**0**	**0**	**0**	**0**
1	**0**	**0**	**0**	**0**	**0**

1. Using the Place Value Chart express the numbers above as powers of ten and without powers of ten.
2. How is the number of zeros in a number related to the exponent for its power of ten?
3. What relationship do you see between place-value positions?
4. $10 \times \square = 240$
5. $10 \times (10 \times 15) =$ ______
6. $10 \times [10 \times (10 \times \square)] = 6{,}000$
7. $10^3 \times \square = 47{,}000$

Lesson 2 – Introductory Task "Dividing into Ten Equal Parts"

Introductory Task	***Guided Practice***	***Collaborative Work***	***Homework***	***Assessment***



1. A large box of cheese sticks holds 1,000 sticks of cheese. Inside the box, the cheese sticks are divided into 10 plastic bags. If each bag has the same number of cheese sticks, how many cheese sticks are in each bag?

2. Next, the cheese sticks in one bag are split equally among 10 containers. How many cheese sticks are in each container?

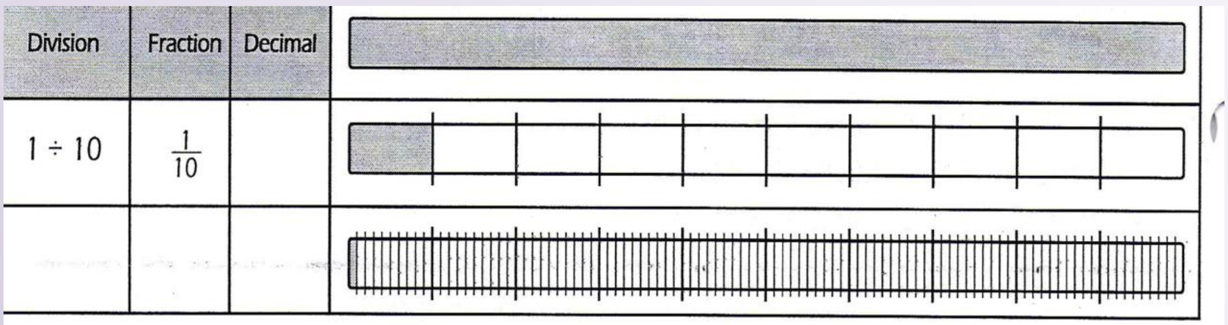
3. Next, each container of cheese sticks is shared equally among 10 students. If each student gets the same number of cheese sticks, how many cheese stick does each student get?

4. Lastly, one cheese stick is split into 10 equal pieces. Each piece is how much of a cheese stick?

5. Write a numerical expression to represent the division by 10 in each problem. Is there more than one way to write this expression?

Lesson 2 – Guided Practice "Dividing into Ten Equal Parts"

1. The top row of the table shows 1 whole stick of cheese. Complete the table to show the amount of cheese represented by the shaded part of the cheese?



Division	Fraction	Decimal	
$1 \div 10$	$\frac{1}{10}$		

2. 1 divided by 10 is equal to one-__________.

3. 0.1 divided by 10 is equal one-___________.

4. If 0.01 of a cheese stick is split into 10 equal parts, how many of those parts would be in one whole stick of cheese?

5. Write a fraction and a decimal to represent the equal parts in Problem 4.

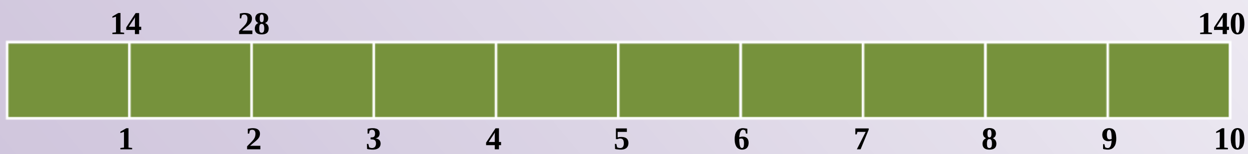
6. 0.01 divided by 10 equal to one-_________.

Lesson 2 – Collaborative Work “Dividing into Ten Equal Parts”

1. Complete the table for each division by 10. Describe any patterns that you find.

Division	Fraction	Whole Number or Decimal
$1000/10$	$\frac{1,000}{10}$	100
100/10	$\frac{100}{10}$	
10/10		
1/10		
0.1/10		
0.01/10		

2. The bar below splits 140 into 10 equal parts. One and two equal parts have been labeled. Label the rest of the equal parts.



3. 270/10 = _____

4. ☐/10 = 1,725

5. (3,5000/10)/10 = _____

6. ((☐/10)/10)/10 = 43

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Lesson 2 – Homework “Dividing into Ten Equal Parts”

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

In the number 55.55, each digit is 5, but the value of each digit is different. Why?

5	**5**	**.**	**5**	**5**

1. Identify the value of 5 in the shaded area of the chart above.

2. How does the value of 5 in the shaded area compare to the 5 on the left explain?

3. How does the value of 5 in the shaded area compare to the 5 on the right? Explain.

Use the chart below to answer the following questions.

Chart A

Hundreds	Tens	Ones	Tenths	Hundredths
				

a. What is the value represented in the place value chart?
b. Use the chart below to generate a number that is 10 times the number represented in above.

Hundreds	Tens	Ones	Tenths	Hundredths

c. Use the chart below to generate a number that is 0.1 times the number represented in Chart A.

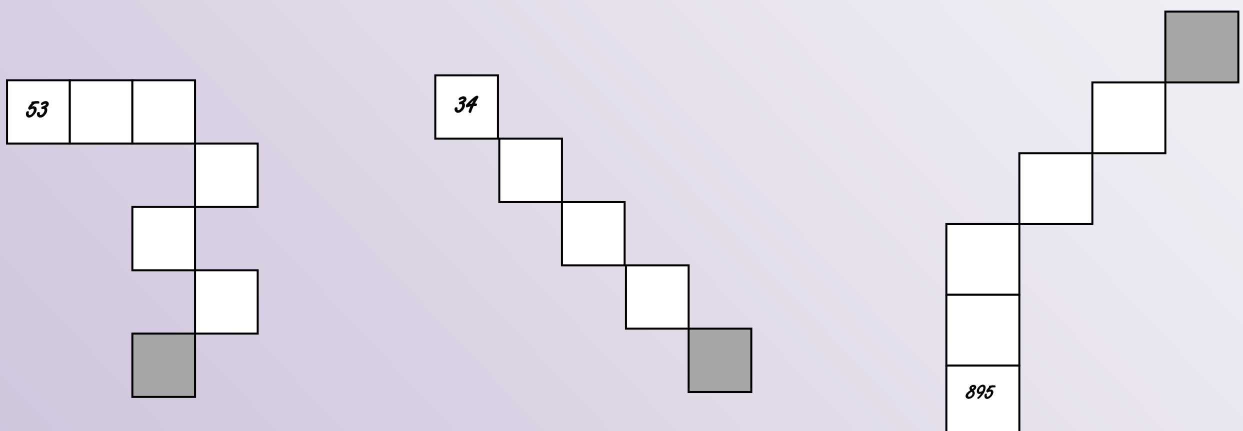
Hundreds	Tens	Ones	Tenths	Hundredths

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Use the chart below to help answer the questions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Shown below are parts of the chart. Without extending the chart, determine which numbers should go in the shaded squares.



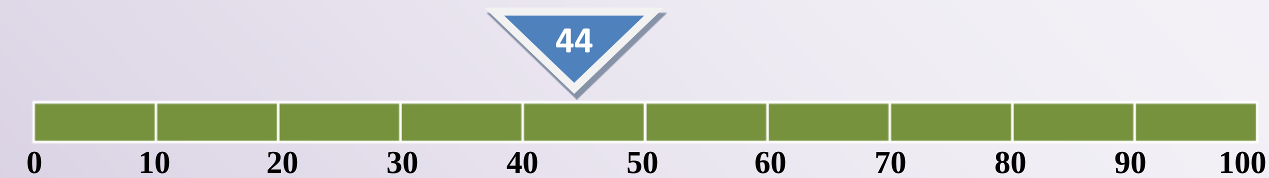
Multiply each shaded value by

- 10 ________ , ________ , ________
- 10^2 ________ , ________ , ________
- 10^3 ________ , ________ , ________
- 10^4 ________ , ________ , ________

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Golden Problem II: Number and Operations in Base Ten (Decimals)

Use the number line below to answer the questions.



The arrow above points to the number 44 on the number line numbered from 0 to 100. On what number will the arrow point when the endpoint of 100 is changed to

- 1,000?
- 100,000?
- 1,000,000?
- 10?
- 1?
- .01?

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Golden Problem I Rubric:

3-Point Response

- The student correctly indicates the following:

95
950
9,500
95,000
950,000
9,500,000

78
780
7,800
78,000
780,000
7,800,000

848
8,480
84,800
848,000
8,480,000
84,800,000

2-Point Response

- The student shows correct work but does not provide the correct answer.
  OR
- The student commits a significant error but provides a correct response based on their incorrect work with clear explanations.
  OR
- The student provides the correct response and shows correct work but fails to provide clear explanations for each part.

1-Point Response

- The student only begins to provide a solution.

0-Point Response

- The response demonstrates insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the required solutions, or the explanation may not be understandable. How decisions were made may not be readily understandable.
  OR
- The student shows no work or justification.

Golden Problem II Rubric:

3-Point Response

- The student correctly indicates the following:

  440
  44,000
  440,000
  4.4
  0.44
  0.0044

2-Point Response

- The student shows correct work but does not provide the correct answer.
  OR
- The student commits a significant error but provides a correct response based on their incorrect work with clear explanations.
  OR
- The student provides the correct response and shows correct work but fails to provide clear explanations for each part.

1-Point Response

- The student only begins to provide a solution.

0-Point Response

- The response demonstrates insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the required solutions, or the explanation may not be understandable. How decisions were made may not be readily understandable.
  OR
- The student shows no work or justification.

5.NBT.5

```
     4, 6 3 5
x        2 7
  ----------
   3 2 4 4 5
   9 4 7 0 0
  ----------
 1 2 7, 1 4 5
```

There is a mistake in the problem above. Identify it and give the most likely reason why it was made.

____

____

____

____

What is the correct answer? ______________________________

5.NBT.5

Using mental computation, will the product of 29 x 28 be over or under 900? Explain how you know.

5.NBT.6

An elementary school has 738 students. The school busses hold 48 students. Every student rides a bus to and from school.

How many busses do they need?

_________________ busses

How do you know your answer is correct?

5.NBT.6

Mrs. Allen needs 60 square tiles to cover the family room floor. The tiles come in boxes of 8.

How many boxes does Mrs. Allen need?

Mrs. Allen needs 65 tiles to cover the basement floor. She will need more boxes for the basement floor than the family room floor. Explain why this is correct.

____

____

____

____

5.NF.3 Converting Fractions of a Unit into a Smaller Unit

a. Five brothers are going to take turns watching their family's new puppy. How much time will each brother spend watching the puppy in a single day if they all watch him for an equal length of time?

Write your answer

- Using only hours,
- Using a whole number of hours and a whole number of minutes, and
- Using only minutes.

b. Mrs. Hinojosa had 75 feet of ribbon. If each of the 18 students in her class gets an equal length of ribbon, how long will each piece be?

Write your answer

- Using only feet,
- Using a whole number of feet and a whole number of inches, and
- Using only inches.

c. Wesley walked 11 miles in 4 hours. If he walked the same distance every hour, how far did he walk in one hour?

Write your answer

- Using only miles,
- Using a whole number of miles and a whole number of feet, and
- Using only feet.

5.NF.3 How Much Pie?

After a class potluck, Emily has three equally sized apple pies left and she wants to divide them into eight equal portions to give to eight students who want to take some pie home.

- Draw a picture showing how Emily might divide the pies into eight equal portions. Explain how your picture shows eight equal portions.
- What fraction of a pie will each of the eight students get?
- Explain how the answer to (b) is related the division problem 3 ÷ 8.

Assessment Check 1: Review

5.NBT.1

Write an odd number that is greater than 10,000 but less than 14,000 on the line.
____

In your head, think number that is 1,000 more than the number you made and write it on the line. ____

Explain your answer.

Tell if the statement in the box is true or false. Then explain your reasoning on the lines below.

500 is ten times larger than 50

____

____

____

____

Which number is equal to 10^4?

a. 100

b. 1,000

c. 10,000

d. 100,000

5.NBT.2

Which is another name for:

$$(7 \times 10^3) + (4 \times 10^2) + (2 \times 10^1) + (9 \times 1)$$

Use what you know about powers of ten and exponents to explain your answer.

5.NBT.5

6 classes at Melanie's school are going on a field trip. Each class has 26 students and 1 teacher. Each bus holds a maximum of 48 people. Her school has asked for 3 buses for the field trip.

Carefully read the following statements:

a. Melanie says that 3 buses are not enough.

b. Melanie says 3 buses will hold a maximum of 144 people.

c. Melanie calculates that 156 people are going on the field trip.

d. The school needs to order one more bus.

Circle the statement above that has an incorrect statement or calculations.

Then write the number that will correct the statement that you chose. _______________

5.NBT.6

Kim got a prize point for every four raffle tickets she sold. She sold 362 raffle tickets.

How many prize points did she get?

In another contest Trey got a prize point for every 3 tickets he sold. Trey sold 271 tickets and got the same number of prize points as Kim. Explain why this is true.

5.NF.3 What is 23 ÷ 5?

a. Jessa has 23 one-dollar bills that she wants to divide equally between her 5 children.

- How much money will each receive? How much money will Jessa have left over?

- Jessa exchanged the remaining one-dollar bills for dimes. If she divides the money equally between her 5 children, how much money will each child get?

b. A website has games available to purchase for $5 each. If Lita has $23, how many games can she purchase? Explain.

c. A jug holds 5 gallons of water. How many jugs can Mark fill with 23 gallons of water? Explain.

d. A class of 23 children will take a field trip. Each car can take 5 children. How many cars are needed to take all the children on the field trip? Explain.

e. Write a division problem for 31 ÷ 4 where the answer is a mixed number. Show how to solve your problem.

New Content

Common Core Standards – Fractions

GRADE 5 FRACTIONS	
5.NF.1 (SLO 1)	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
5.NF.2 (SLO 2)	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
5.NF.4 a (SLO 3)	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) x q as ***a*** parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b. For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x 4 = 8/15. (In general, (a/b) x (c/d) = ac/bd.)
5.NF.5 a,b (SLO 4)	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n*×*a*)/(*n*×*b*) to the effect of multiplying *a*/*b* by 1.
	Standard below will be carried over into Unit 4
5.NF.6 (SLO 5)	Solve real world problems involving multiplication of fractions and mixed numbers, e.g. by using visual fractions models or equations to represent the problem.
5.NF.7 a-c (SLO 6)	**Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions** a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Essential Concepts

- Equivalent fractions can be created by multiplying the fraction $\frac{a}{b} \times \frac{n}{n}$ where $\frac{n}{n} = 1$
- Fractions with unlike denominators can be added and subtracted by creating and using equivalent fractions. This is determined by subdividing (i.e., further dividing a fractional part) the fraction of one using the denominator of other $i.e.$ ($\frac{a}{b} + \frac{c}{d} = \frac{a \times d}{b \times d} + \frac{c \times b}{d \times b} = \frac{ad+bc}{bd}$)

  Note: Subdividing is actually the process of multiplying a fractional part by a whole that will make each fractional part smaller.

- Benchmark fractions and number sense can be used to determine if a solution is reasonable.
- Fractions represent division of the numerator by the denominator: $\frac{a}{b} = a \div b$.
- Division problems involving whole numbers and fractions may be represented and solved using visual fraction models.
- Multiplication can be interpreted as scaling (resizing).
- The relationship between the size of the factors and the size of the product can be interpreted without solving for the product:
- Multiplying a given number by a fraction less than 1, results in a product smaller than the given number; likewise, multiplying a given number by a fraction greater than 1, results in a product greater than the given number.

Essential Questions

- Why is it important to estimate before solving problems?
- How can you mentally estimate the sum or difference of fractions with unlike denominators?
- Explain why multiplying a fraction by $\frac{n}{n}$ does not change the value of the original fraction.
- Compare and contrast how fraction models, benchmark fractions and equivalent fractions can be used to solve addition and subtraction of fractions with unlike denominators
- How are fractions related to division?
- Write a multiplication or division story problem and give the fraction that can be used to represent and solve your story.
- Use a model to explain why multiplying a number by a fraction less than 1 results in a product smaller than the given number.
- How is multiplication similar to or different from scaling (resizing)?
- How is dividing a whole number by a fraction similar to/different from dividing a fraction by a whole number?

Connections to the Mathematical Practices

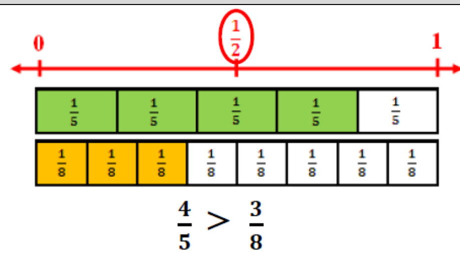
1	Make sense of problems and persevere in solving them
	Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2	Reason abstractly and quantitatively
	Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3	Construct viable arguments and critique the reasoning of others
	In **fifth** grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4	Model with mathematics
	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5	Use appropriate tools strategically
	Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.
6	Attend to precision
	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7	Look for and make use of structure
	In **fifth grade**, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
8	Look for and express regularity in repeated reasoning
	Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Vocabulary

Visual Definition

The terms below are for teacher reference only and are not to be memorized by students. Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

benchmark fractions



Fractions that are commonly used for estimation. A benchmark fraction helps you compare two fractions.

common denominator

12 is a common denominator for $\frac{2}{3}$ and $\frac{3}{4}$

For two or more fractions, a common denominator is a common multiple of the denominators.

common multiple

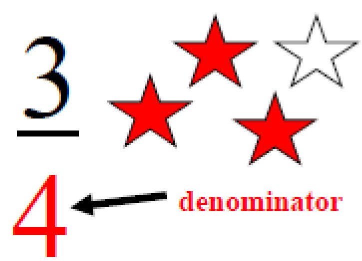
4, 8, 12, 16, 20, 24, 28, 32, 36…

6, 12, 18, 24, 30, 36, 42…

Common Multiples of 4 and 6:
12, 24, 36…

Any common multiple of two or more numbers.

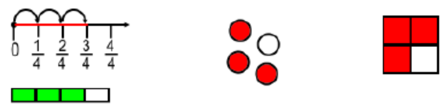
denominator

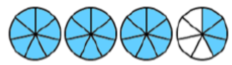
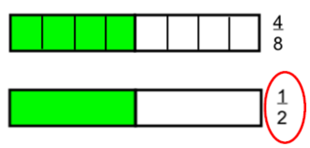


The quantity below the line in a fraction. It tells the number of equal parts into which a whole is divided.

equivalent fractions

Fractions that have the same value.

fraction	Measurement Model / Set Model / Area Model  Bar Diagram (thickened number line)	A way to describe a part of a whole or a part of a group by using equal parts.
fraction bar	$\frac{2}{3} = 2 \div 3$	A horizontal bar that separates the numerator and the denominator.
fraction greater than one	$\frac{7}{5}$ ← greater than denominator	A fraction where the numerator is greater than the denominator.
fraction less than one	 $\frac{5}{8}$ ← less than the denominator	A fraction less than one. In a proper fraction the numerator is less than the denominator.
like denominators	$\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$	Denominators in two or more fractions that are the same.
lowest terms	 $\frac{4}{8}$ in lowest terms is $\frac{1}{2}$.	A fraction where the numerator and denominator have no common factor greater than 1.

	Example:	
mixed number	$3\frac{3}{7}$ 	A number with an integer and a fraction part.
numerator	$\frac{4}{5}$ ← numerator 	The number or expression written above the line in a fraction.
simplest form	 **A fraction in simplest form has the fewest possible pieces.**	A fraction is in simplest form when the greatest common factor of the numerator and denominator is 1.
simplify		To express a fraction in simplest form.
unit fraction	$\frac{1}{2}$ **Example** 	A fraction with a numerator of 1. A unit fraction names 1 equal part of a whole.
unlike denominators	$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$	Denominators that are not equal.

Potential Student Misconceptions

- **Students may not understand that fractional parts must be equal amounts.**
  Students may create fractions of circular regions by dividing them horizontally and vertically, creating unequal parts. Having students draw fractional models. Ask questions that focus students on the equality of parts. Having students cut out the parts and place them on top of each other may highlight this idea.

- **Students may not realize that, in order to compare fractions with models, the wholes must be the same size.**
  From time to time have students draw their own representations of the fractions they are comparing. Always giving students pre-made physical models can mask this misconception. Have students solve this problem: Fernando had ½ of a pizza and Lucy had 1/3 of a pizza. Lucy said that she had more pizza. Is she correct? If so, how could that be? If not, why not?

- **Students may mix models when adding, subtracting, or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths.**
  Remind students that the representations need to be from the same whole.

- **Students may believe that multiplication always results in a larger number. Students may also believe that division always results in a smaller number.**
  Connect the meaning of multiplication and division of fractions with whole number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division. Using models when multiplying with fractions will enable students to see that the results will be smaller. Using models when dividing with fractions will enable students to see that the results will be larger.

Common Core Standards: 5.NF.1 & 2

Use equivalent fractions as a strategy to add and subtract fractions

Goal:

Students develop an understanding of and fluency with addition and subtraction of mixed numbers and fraction with unlike denominators. They conclude that common denominators are useful for adding and subtracting fractions. They estimate sums and differences of fractions when assessing the reasonableness of results.

Prerequisite Skills:

- Understand the concept of equivalent fractions
- Understand that fractions can be parts of a whole, set, or points of distance on a number line
- Add/subtract with like denominators
- Multiply and divide
- Determine LCM

Essential Questions:

What generalizable procedures used for whole numbers can be applied to adding and subtracting fractions and mixed numbers?

Which models/representations can be used to improve the understanding of the addition and subtraction of fractions and mixed numbers?

How can benchmark fractions be used to estimate sums and differences of fractions and mixed numbers and to assess the reasonableness of results?

How is the addition and subtraction of fractions and mixed numbers used to solve real-world and mathematical problems including those involving measurement, geometry and data?

Embedded Mathematical Practices

MP.1 Make sense of problems and persevere in solving them

MP.2 Reason abstractly and quantitatively

MP.3 Construct viable arguments and critique the reasoning of others

MP.4 Model with mathematics

LESSON 4
5.NF.1-2
Golden Problem

LESSON 3
5.NF.2 (Extension)
Solve word problems involving addition & subtraction of fractions

LESSON 2
5.NF.2
Solve word problems involving addition & subtraction of fractions; Emphasis on reasonableness

LESSON 1
5.NF.1
Add and subtract fractions w/unlike denominators (including mixed numbers)

Lesson Structure:
Introductory Task
Guided Practice
Collaborative Work
Journal Questions
Skill Building
Homework

Lesson 1: Introductory Tasks

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

PREREQUISITE COMPENTENCIES FOR THIS TASK

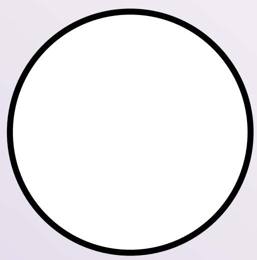
- Understand the concept of and generate equivalent fractions
- Understand that the size of a fractional part is relative to the size of the whole
- Understand the relationship between the size of the denominator and the size of the parts, i.e., as the denominator gets larger the size of the parts gets smaller
- Locate fractions, including improper fractions, and mixed numbers on a number line
- Use models to show addition and subtraction of fractions with like denominators and generate a rule for addition and subtraction of fractions with like denominators

Introductory Tasks

5.NF.1

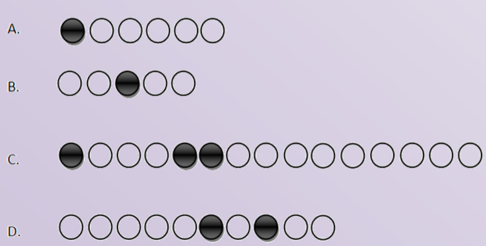
Add and subtract fractions of unlike denominators by replacing given fractions with equivalent fractions.

Problem 1: Use one color to show $\frac{1}{8} + \frac{3}{4}$.



Problem 2: Use either a visual model to show how many $\frac{1}{6}$'s are in $5\ \frac{2}{3}$.

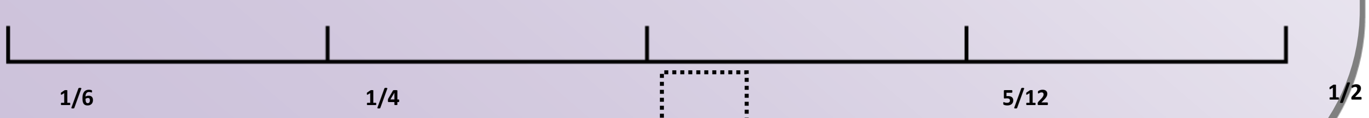
Problem 3: Mario shaded $\frac{1}{5}$ of the shapes in a group. Which of the following could be Mario's group?



Focus Question:

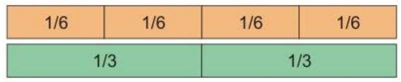
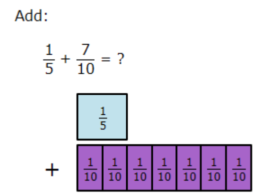
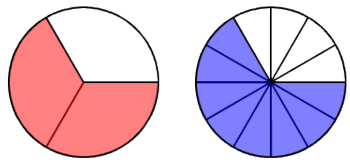
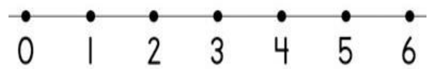
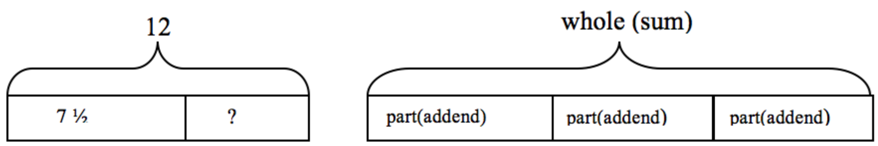
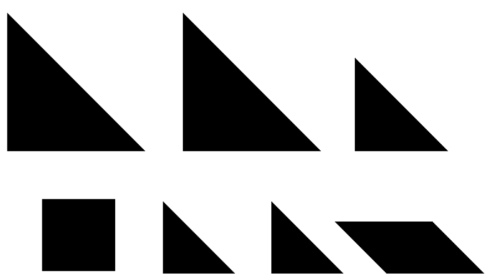
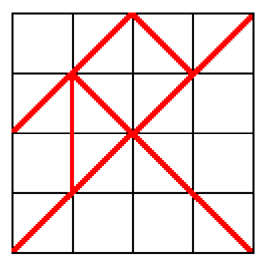
When is finding 'like' denominators useful?

Problem 4: What is the fraction in the box? Express your answer in the simplest form.



Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

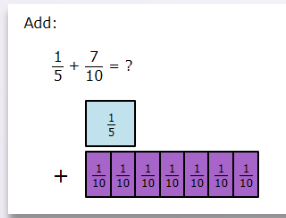
Multiple Representations Framework

Concrete and Pictorial Representations	
Equal Partitioning and Unitizing ***Using Visual Fraction Models*** • **Fraction Strips** • **Fraction Circles** • **Number line**	   
Bar Model 	*Leticia read 7 ½ books for the read-a-thon. She wants to read 12 books in all. How many more books does she have to read?*  12 - 7 ½ = ? or 7 ½ + ? = 12 so Leticia needs to read 4 ½ more books.
Tangram Puzzle Choosing each piece of the Tangram set, students are asked identify the size of the pieces based upon • The original square • The size of a select piece • When assigning a value to each piece, for example when the large right triangle is equal to 2.	 
Equivalent Fractions	*For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
Benchmark Fractions	**1/2, 1/3, 1/4, 1/5, 1/6. 1/8, 1/10**
Abstract Representations	
Basic Mathematical Properties	**Additive Inverse**

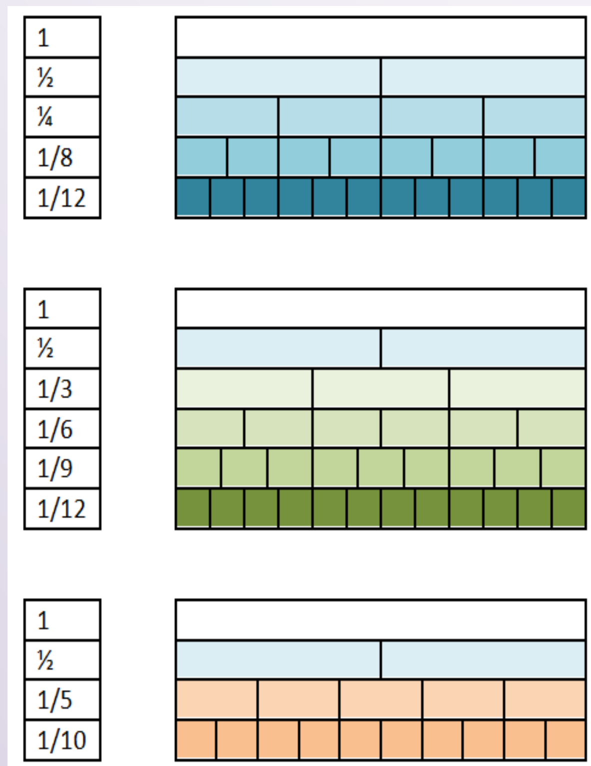
	Example: 7 + (-7) = 0
Algorithm	In general, **a/b+c/d=(ad+bc)/bd**

Guided Practice

Use a model to solve the following. For example:

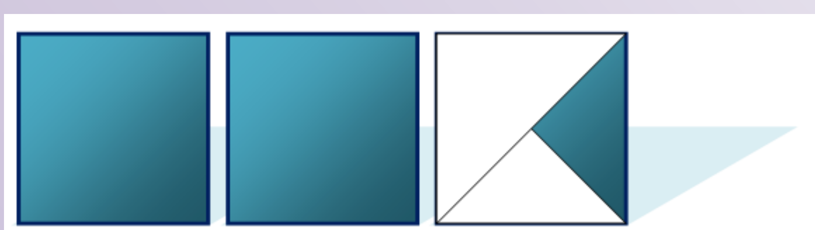


1. $\frac{1}{2} + \frac{1}{3}$
2. $\frac{3}{4} + \frac{1}{6}$
3. $\frac{5}{6} + \frac{2}{3}$
4. $1\frac{2}{3} + \frac{5}{4}$
5. $1\frac{5}{6} - \frac{2}{9}$
6. $\frac{3}{4} - \frac{5}{12}$
7. $\frac{8}{9} + \frac{1}{6}$
8. $\frac{7}{8} - \frac{1}{4}$



See Appendix

9. **Below are three similar squares. What fraction of the squares is NOT shaded?**

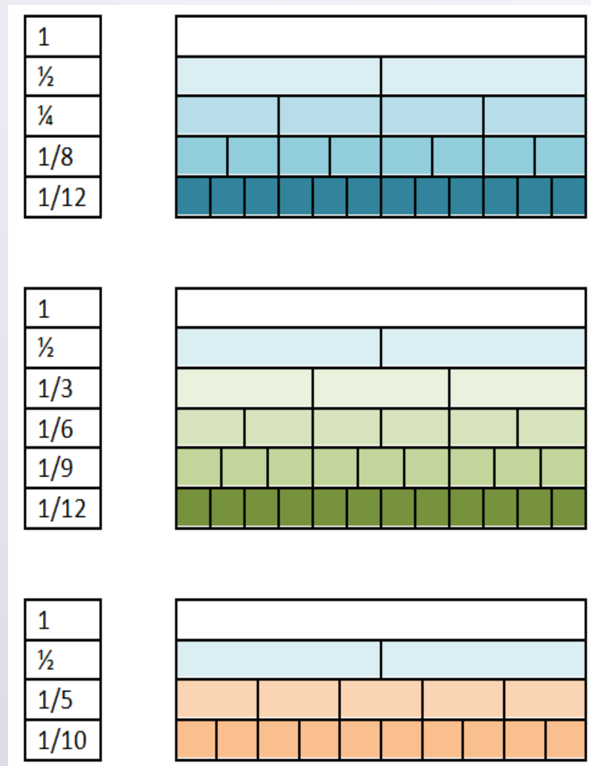


Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Lesson 1 – Collaborative Work

Use a model to solve the following.

1. $\frac{1}{2} - \frac{1}{3}$
2. $\frac{3}{4} + \frac{1}{2}$
3. $\frac{7}{12} + \frac{1}{3}$
4. $\frac{1}{8} + \frac{3}{4}$
5. $1\frac{5}{6} - \frac{2}{9}$
6. $\frac{4}{6} - \frac{1}{8}$
7. $\frac{6}{6} + \frac{1}{3}$
8. $\frac{3}{8} - \frac{1}{4}$



9. Mrs. Day baked apple pie for her family. The boys ate ½ of the pie and the girls ate ¼ of what was left. How much pie is in the pan?
10. Darnay filled a measuring cup with ¼ of a cup of vegetable oil. Then he poured 1/8 of a cup of the oil into a frying pan. How much oil is left in the measuring cup?

Journal Question: *Explain why the equation 1/7 +2/7 = 3/14 is incorrect.*

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Lesson 1 – Homework

Use a model to solve the following.

1. $\frac{7}{12} - \frac{1}{4}$

2. $\frac{1}{6} + \frac{1}{2}$

3. $\frac{3}{5} + \frac{1}{2}$

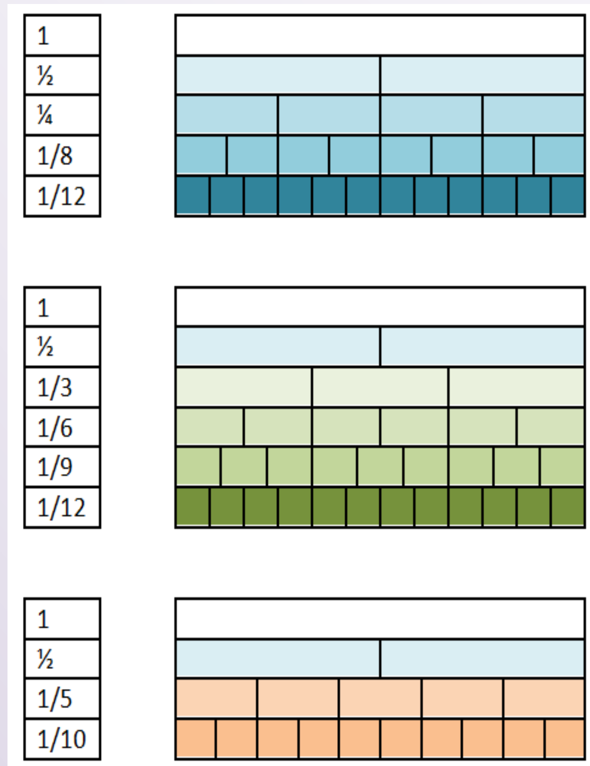
4. $\frac{3}{4} + \frac{1}{6}$

5. $4\frac{1}{3} - \frac{1}{6}$

6. $\frac{5}{6} - \frac{1}{4}$

7. $\frac{6}{6} + \frac{1}{3}$

8. $5\frac{3}{8} + 2\frac{1}{4}$



9. In a pie eating contest Gary ate 3/5 of the pie in 30 seconds then ½ of what remained in the next 30 seconds. How much did he eat all together?

10. Keisha filed a bucket with 5/8 of a gallon of water. Later she poured out ¼ of a gallon of the water. How much water is left in the bucket?

5.NF. 2 Lesson 2: Introductory Task – *Making Muffins*

Introductory Task	**_Guided Practice_**	**_Collaborative Work_**	**_Homework_**	**_Assessment_**

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Introductory Task

Each year the Williams family makes a mark on the wall to keep track of their son's height. At age 11 he grew 1/10 of an inch and when he was 12, he grew 4/5 of an inch. In total, how much did their son grow in the 2 years? Check the reasonableness of your answer.

Multiple Representations

- Use of benchmark fractions
- Visual Fraction Models
- Bar Models
- Equivalent Fractions

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Lesson 2 – Guided Practice

Introductory Task	*Guided Practice*	*Collaborative Work*	*Homework*	*Assessment*

Guided Practice

1. Which has the smallest value? Explain your reasoning.

   1/3 +/4

   ¼ + 1/5

   1/5 + 1/6

   ½ + 1/3

2. Maria received a chocolate chip cookie as big as a birthday cake for a present. She cut it into $\frac{1}{6}$'s and shared the cookie with her friend LeAnna. Maria ate $\frac{3}{6}$ the cookie and Leanna ate $\frac{1}{3}$. Together, how much did they eat?

3. Martin was making play dough. He added ¾ cup of flour to the bowl. Then he added another 3/6 cup. Is the total amount of flour he used greater or less than one? How much flour did he use?

4. Marty divided a candy bar into 12 equal parts. He ate 1/6 of the candy bar before lunch. He ate 1/4 of the candy bar after lunch. Did he eat more or less than 1-half of the candy bar? Did he eat the whole candy bar? Explain your reasoning.

5. Terri ate $\frac{5}{6}$ of a small pizza and $\frac{11}{12}$ of another small pizza. Did she eat more than one whole pizza? Explain your reasoning.

6. Alex used 1/3 cup of flour in one recipe and ¼ cup of flour in another recipe. Together did he use more than ½ cup of flour? Explain your reasoning.

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Collaborative Work

1. List 5 fractions greater than $\frac{1}{2}$. How do you know that they are greater than $\frac{1}{2}$?

2. Terri ate $\frac{1}{2}$ of a small pizza and $\frac{5}{12}$ of another small pizza. About how much of a whole pizza did she eat? With your models, find out the exact amount.

3. Allie rode her bicycle 7/8 of a mile to school. Then she rode ¼ of a mile to her friend's house. About how far did she ride altogether? Exactly how far did she ride altogether?

4. Because of a rainstorm, the water level in a swimming pool rose $\frac{2}{3}$ of an inch. The following day it rained again. The pool rose another $\frac{11}{12}$ of an inch. About how high did the water level increase?

5. Karen had 4 $\frac{3}{8}$ yd. of cotton fabric. She used 3 $\frac{5}{8}$ for a skirt. How much fabric was left?

6. In Mr. Mark's class, $\frac{1}{2}$ of the students are making videos for their project and $\frac{1}{10}$ are making dioramas. What fraction of the students are making either a video or a diorama?

7. In the summer planet Moo-Noo is $\frac{7}{8}$ of a light year away from planet earth. In the winter it is $\frac{3}{4}$ away from earth. How much farther away is planet Moo-Noo in the summer?

Journal Question

If you ran 3/ 4 of a mile before lunch and ran 7/8 of a mile after lunch. How many miles did you run? In a letter, explain how another student can check the reasonableness of your answer

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Lesson 2 – Homework

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

1. Marissa placed an order for 3/4 of a sack of brown lentils and 1/2 of a sack of green lentils. How much more brown lentils did Marissa order?

2. Yardley's zoo has two elephants. The male elephant weighs 3/5 of a ton and the female elephant weighs 3/10 of a ton. How much more does the male weigh than the female?

3. In Charles's apartment complex, 1/10 of the apartments are one-bedroom apartments and 1/2 are two-bedroom apartments. What fraction of the apartments are either one- or two-bedroom apartments?

4. Sandra bought 2¾ yards of red fabric and 1¼ of blue. How much cloth did she buy in all?

5. Emma made ½ a quart of hot chocolate. Each mug holds 1/10 of a quart. How many mugs will she be able to fill?

6. Yaira owns 7 acres of farm land. She plants 5/6 of the land with rye seed. How many acres did she plant with rye?

7. Jessica bought 8/9 of a pound of chocolates and ate 1/3 of a pound. How much was left?

8. Tom bought a board that was 7/8 of a yard long. He cut off 1/2 of a yard. How much was left?

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

5.NF. 2 Lesson 3: (Extension)

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. (Extended Problem Solving)

Introductory Task

Problem 1: If $2\frac{8}{10} = 1\frac{n}{5}$, then how can you find n?

Problem 2: The track is 3/5 of a mile long. If Tyrone jogged around it twice, how far did he run?*

* Encourage the use of repeated addition in examples such as these. Module 2 explores the multiplication and division of fractions.

Multiple Representations

- Use of benchmark fractions
- Visual Fraction Models
- Bar Models
- Equivalent Fractions

Lesson 3 – Guided Practice

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

Guided Practice

1. Masayo's Cheese Shop sells a variety platter with 11/12 of a pound of Cheddar cheese, 5/12 of a pound of Swiss cheese, and 1/12 of a pound of Muenster cheese. How many pounds of cheese are on the platter?

2. During a canned food drive, items were sorted into bins. The drive resulted in 3/5 of a bin of soup, 3/5 of a bin of vegetables, and 4/5 of a bin of pasta. Altogether, how many bins would the canned food take up?

3. Which apple weighs more, one that weighs 2/3 of a pound or one that weighs 5/6 of a pound?

4. Tim's fish tank was filled with 4/9 liters of water. He added more water. Now it is 2/3 full. How much water did he add?

5. Find the value of $6 - 3\frac{4}{9}$.

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Collaborative Work

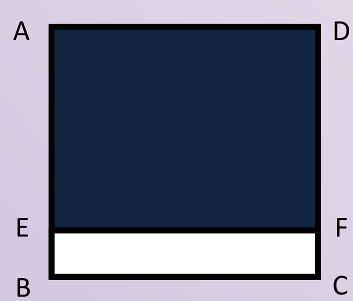
1. An equilateral triangle measures 3½ inches on one side. What is the perimeter of the triangle?

2. Frank brought 20 cupcakes to school. He and his friends ate 3/4 of the cupcakes before lunch. Frank decided to give his teacher 2/5 of the remaining cupcakes. How many cupcakes did Frank give his teacher?

3. A metal company makes sheets of metal that are 1/5 of an inch thick. If a worker makes a stack of 10 sheets, how many inches thick will the stack be?

4. Jon spent 4/9 of his savings on pants and 1/3 on 2 shirts how much of his savings does he have left?

5. ABCD is a square of side 50 mm. The area of BCEF is $\frac{1}{5}$ of the area of ABCD. Find the area of the shaded part.



Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Lesson 3 – Homework

Introductory Task	*Guided Practice*	*Collaborative Work*	*Homework*	*Assessment*

1. Stanley ordered two pizzas cut into eighths. If he ate 5/8 of a pizza, how much was left?

2. If 3/5 of a number is 24, what is the number?

3. Find the value of 4 ½ minus 1 2/3?

4. Travis completed ¾ of his trip by plane, and the remaining distance by car. How far did he travel by car?

5. Jen uses ¾ cup of butter for every 1 batch of cookies that she bakes. How many cups of butter will Jen use when she bakes 6 batches of cookies?

Source: Problem(s) generated by the Office of Mathematics and/or adapted from various web resources.

Lesson 4 – Golden Problem

Introductory Task	Guided Practice	Collaborative Work	Homework	Assessment

Dante decided to spend Saturday at home and catching up on watching all of his favorite shows that he had missed throughout the week. Because he recorded all of the programs, sometimes he would 'skip" the commercials and fast forward to the best parts of each show.

- First, he watched two comedies. He spent $\frac{1}{3}$ of an hour watching the first comedy and ¾ of an hour watching the second comedy. About how much time did he spend in total watching the comedies?
- Dante recorded a total of 7 $\frac{5}{8}$ hours of television. 1 $\frac{1}{4}$ of that time was commercials. If Dante erases all of the commercials, how much time would be left of his recordings?
- Dante spent about $\frac{3}{5}$ of an hour watching his favorite cartoon. He liked it so much that he watched it a total of 3 times. How much time did he spend watching his favorite cartoon?
- Dan was watching a musical that lasted 2 $\frac{1}{2}$ hours and ended at 7pm. What time did he begin watching the musical?

Golden Problem RUBRIC

SCORE POINT = 3

The student correctly determines that Dante spent **approximately 1 hour** watching the comedies.

AND

correctly determines that without commercials, his recordings are **6 $\frac{3}{8}$ hours** long.

AND

correctly determines that he watched his favorite cartoon for a total o f **$\frac{9}{5}$ or 1 $\frac{4}{5}$ hours.**

AND

correctly determines that Dante began watching the musical at **4:30pm.**

SCORE POINT = 2

The student correctly solves at least two of the four parts. Explanation or steps must be shown on how the student arrives at the answers. The explanation may not be clear.

SCORE POINT = 1

The student correctly solves one of the four parts. However, the student shows incomplete explanation.

SCORE POINT = 0

The response shows insufficient understanding of the problem's essential mathematical concepts.
The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation.

Golden Problem_Basic Answer Key:

Part I:	**Approximately 1 hour**
Part II:	**$6\frac{3}{8}$ hours**
Part III:	**$\frac{9}{5}$ or $1\frac{4}{5}$ hours**
Part IV (BONUS):	**4:30pm**

Assessment Checks

Assessment Check 2: New Content – Please use the recommended resources below for formative assessment options.

Associated Illustrative Math Tasks

Measuring Cups (5.NF.A)

Lucy has measuring cups of sizes 1 cup, $\frac{1}{2}$ cup, $\frac{1}{3}$ cup, and $\frac{1}{4}$ cup. She is trying to measure out $\frac{1}{6}$ of a cup of water and says "if I fill up the the $\frac{1}{2}$ cup and then pour that into the $\frac{1}{3}$ cup until it is full, there will be $\frac{1}{6}$ of a cup of water left."

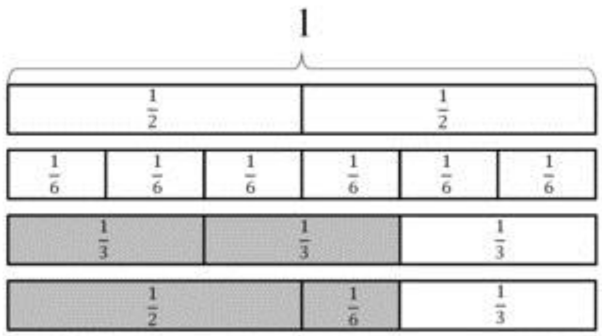
a. Is Lucy's method to measure $\frac{1}{6}$ of a cup of water correct? Explain.

b. Lucy wonders what other amounts she can measure. Is it possible for her to measure out $\frac{1}{12}$ of a cup? Explain.

c. What other amounts of water can Lucy measure?

Egyptian Fractions (5.NF.A.1)

Ancient Egyptians used unit fractions, such as $\frac{1}{2}$ and $\frac{1}{3}$, to represent all fractions. For example, they might write the number $\frac{2}{3}$ as $\frac{1}{2}+\frac{1}{6}$.



We often think of $\frac{2}{3}$ as $\frac{1}{3}+\frac{1}{3}$, but the ancient Egyptians would not write it this way because they didn't use the same unit fraction twice.

a. Write each of the following Egyptian fractions as a single fraction:
   i. $\frac{1}{2}+\frac{1}{3}$,
   ii. $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}$,
   iii. $\frac{1}{4}+\frac{1}{5}+\frac{1}{12}$

b. How might the ancient Egyptians have written the fraction we write as $\frac{3}{4}$?

Mixed Numbers with Unlike Denominators (5.NF.A.1)

Find two different ways to add these two numbers:

$$1\frac{1}{3} + 2\frac{3}{5}$$

Jog-A-Thon (5.NF.A.1)

Alex is training for his school's Jog-A-Thon and needs to run at least 1 mile per day. If Alex runs to his grandma's house, which is $\frac{5}{8}$ of a mile away, and then to his friend Justin's house, which is $\frac{1}{2}$ of a mile away, will he have trained enough for the day?

Finding Common Denominators to Add (5.NF.A.1)

a. To add fractions, we usually first find a common denominator.
   i. Find two different common denominators for $\frac{1}{5}$ and $\frac{1}{15}$.
   ii. Use each common denominator to find the value of $\frac{1}{5}+\frac{1}{15}$. Draw a picture that shows your solution.
b. Find $\frac{3}{4}+\frac{1}{5}$. Draw a picture that shows your solution.
c. Find $\frac{14}{8}+\frac{15}{12}$.

Finding Common Denominators to Subtract (5.NF.A.1)

a. To subtract fractions, we usually first find a common denominator.
   i. Find two different common denominators for $\frac{1}{2}$ and $\frac{1}{14}$.
   ii. Use each common denominator to find the value of $\frac{1}{2}-\frac{1}{14}$. Draw a picture that shows your solution.
b. Find $\frac{5}{9}-\frac{1}{6}$. Draw a picture that shows your solution.
c. Find $\frac{21}{10}-\frac{24}{15}$.

Making S'Mores (5.NF.A.1)

Nick and Tasha are buying supplies for a camping trip. They need to buy chocolate bars to make s'mores, their favorite campfire dessert. Each of them has a different recipe for their perfect s'more.

Nick likes to use $\frac{1}{2}$ of a chocolate bar to make a s'more.

Tasha will only eat a s'more that is made with exactly $\frac{2}{5}$ of a chocolate bar.

a. What fraction of a chocolate bar will Nick and Tasha use in total if they each eat one s'more?
b. Nick wants to cut one chocolate bar into pieces of equal size so that he and Tasha can make their s'mores. How many pieces should he cut the chocolate bar into so that each person will get the right amount of chocolate to make their perfect s'more?
c. After Nick cuts the chocolate bar into pieces of equal size, how many pieces of the chocolate bar should he get? How many pieces of the chocolate bar should he give to Tasha?

Do These Add Up? (5.NF.A.2)

For each of the following word problems, determine whether or not $\left(\frac{2}{5}+\frac{3}{10}\right)$ represents the problem. Explain your decision.

1. A farmer planted $\left(\frac{2}{5}+\frac{3}{10}\right)$of his forty acres in corn and another $\frac{3}{10}$ of his land in wheat. Taken together, what fraction of the 40 acres had been planted in corn or wheat?
2. Jim drank $\frac{2}{5}$ of his water bottle and John drank $\frac{3}{10}$ of his water bottle. How much water did both boys drink?
3. Allison has a batch of eggs in the incubator. On Monday $\frac{2}{5}$ of the eggs hatched, By Wednesday, $\frac{3}{10}$ more of the original batch hatched. How many eggs hatched in all?
4. Two fifths of the cross-country team arrived at the weight room at 7 a.m. Ten minutes later, $\frac{3}{10}$ of the team showed up. The rest of the team stayed home. What fraction of the team made it to the weight room that day?
5. Andy made 2 free throws out of 5 free throw attempts. Jose made 3 free throws out of 10 free throw attempts. What is the fraction of free throw attempts that the two boys made together?
6. Two fifths of the students in the fifth grade want to be in the band. Three tenths of the students in the fifth grade want to play in the orchestra. What fraction of the students in the fifth grade want to be in one of the two musical groups?
7. There are 150 students in the fifth grade in Washington Elementary School. Two fifths of the students like soccer best and $\frac{3}{10}$ of them like basketball best. What fraction like soccer or basketball best?
8. The fifth grade at Lincoln School has two mixed-sex soccer teams, Team A and Team B. If $\frac{2}{5}$ of Team A are girls and $\frac{3}{10}$ of Team B are girls, what fraction of the players from the two teams are girls?
9. Wesley ran $\frac{2}{5}$ of a mile on Monday and $\frac{3}{10}$ of a mile on Tuesday. How far did he run those two days?

To Multiply or Not to Multiply? (5.NF.A)

Some of the problems below can be solved by multiplying $\frac{1}{8}\times\frac{2}{5}$, while others need a different operation. Select the ones that can be solved by multiplying these two numbers. For the remaining, tell what operation is appropriate. In all cases, solve the problem (if possible) and include appropriate units in the answer.

a. Two-fifths of the students in Anya's fifth grade class are girls. One-eighth of the girls wear glasses. What fraction of Anya's class consists of girls who wear glasses?
b. A farm is in the shape of a rectangle $\frac{1}{8}$ of a mile long and $\frac{2}{5}$ of a mile wide. What is the area of the farm?
c. There is $\frac{2}{5}$ of a pizza left. If Jamie eats another $\frac{1}{8}$ of the original whole pizza, what fraction of the original pizza is left over?
d. In Sam's fifth grade class, $\frac{1}{8}$ of the students are boys. Of those boys, $\frac{2}{5}$ have red hair. What fraction of the class is red-haired boys?
e. Only $\frac{1}{20}$ of the guests at the party wore both red and green. If $\frac{1}{8}$ of the guests wore red, what fraction of the guests who wore red also wore green?
f. Alex was planting a garden. He planted $\frac{2}{5}$ of the garden with potatoes and $\frac{1}{8}$ of the garden with lettuce. What fraction of the garden is planted with potatoes or lettuce?
g. At the start of the trip, the gas tank on the car was $\frac{2}{5}$ full. If the trip used $\frac{1}{8}$ of the remaining gas, what fraction of a tank of gas is left at the end of the trip?
h. On Monday, $\frac{1}{8}$ of the students in Mr. Brown's class were absent from school. The nurse told Mr. Brown that $\frac{2}{5}$ of those students who were absent had the flu. What fraction of the absent students had the flu?
i. Of the children at Molly's daycare, $\frac{1}{8}$ are boys and $\frac{2}{5}$ of the boys are under 1 year old. How many boys at the daycare are under one year old?
j. The track at school is $\frac{2}{5}$ of a mile long. If Jason has run $\frac{1}{8}$ of the way around the track, what fraction of a mile has he run?

Salad Dressing (5.NF.A.2)

Aunt Barb's Salad Dressing Recipe
• $\frac{1}{3}$ cup olive oil • $\frac{1}{6}$ cup balsamic vinegar • a pinch of herbs • a pinch of salt Makes 6 servings

1. How many cups of salad dressing will this recipe make? Write an equation to represent your thinking. Assume that the herbs and salt do not change the amount of dressing.

2. If this recipe makes 6 servings, how much dressing would there be in one serving? Write a number sentence to represent your thinking.

Painting a Wall (5.NF.B)

Nicolas is helping to paint a wall at a park near his house as part of a community service project. He had painted half of the wall yellow when the park director walked by and said,

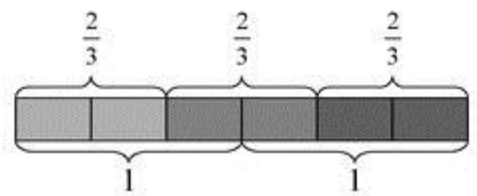
This wall is supposed to be painted red.

Nicolas immediately started painting over the yellow portion of the wall. By the end of the day, he had repainted $\frac{5}{6}$ of the yellow portion red.

What fraction of the entire wall is painted red at the end of the day?

Connor and Makayla Discuss Multiplication (5.NF.B.4)

Makayla said, "I can represent $3 \times \frac{2}{3}$ with 3 rectangles each of length $\frac{2}{3}$."

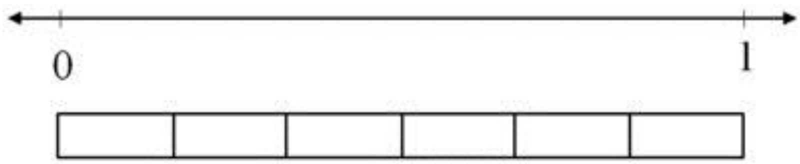


Connor said, "I know that $\frac{2}{3} \times 3$ can be thought of as $\frac{2}{3}\ of\ 3$. Is 3 copies of $\frac{2}{3}$ the same as $\frac{2}{3}\ of\ 3$?"

1. Draw a diagram to represent $\frac{2}{3}\ of\ 3$.
2. Explain why your picture and Makayla's picture together show that $3 \times \frac{2}{3} = \frac{2}{3} \times 3$.
3. What property of multiplication do these pictures illustrate?

Folding Strips of Paper (5.NF.B.4)

a. Label the points on the number line that correspond to $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, and\ \frac{5}{6}$.



b. Carefully cut out a strip of paper that has a length of $\frac{5}{6}$.
   i. Bring the ends of the strip together to fold the strip of paper in half. How long is half of the strip? Use your strip to mark this point on the number line.
   ii. What two numbers can you multiply to find the length of half the strip? Write an equation to show this.

c. Unfold your paper strip so that you start with $\frac{5}{6}$ again. Now fold the strip of paper in half and then in half again.
   i. How long is half of half of the strip? Use your strip to mark this point on the number line.
   ii. What numbers can you multiply to find the length of half the strip? Write an equation to show this.

Running a Mile (5.NF.B.5)

Curt and Ian both ran a mile. Curt's time was $\frac{8}{9}$ Ian's time. Who ran faster? Explain and draw a picture.

Fundraising (5.NF.B.5)

Cai, Mark, and Jen were raising money for a school trip.

- Cai collected $2\frac{1}{2}$ times as much as Mark.
- Mark collected $\frac{2}{3}$ as much as Jen.

Who collected the most? Who collected the least? Explain.

Calculator Trouble (5.NF.B.5)

Luke had a calculator that will only display numbers less than or equal to 999,999,999. Which of the following products will his calculator display? Explain.

a. $792 \times 999{,}999{,}999$

b. $\frac{1}{2} \times 999{,}999{,}999$

c. $\frac{15}{4} \times 999{,}999{,}999$

d. $0.67 \times 999{,}999{,}999$

Comparing a Number and a Product (5.NF.B.5)

Decide which number is greater without multiplying.

1. $817 \; or \; 235 \times 817$

2. $99 \; or \; \frac{1}{4} \times 99$

3. $\frac{51}{100} \; or \; \frac{51}{100} \times 301$

4. $\frac{13}{90} \; or \; \frac{2}{3} \times \frac{13}{90}$

5. $\frac{101}{102} \; or \; \frac{101}{102} \times \frac{101}{102}$

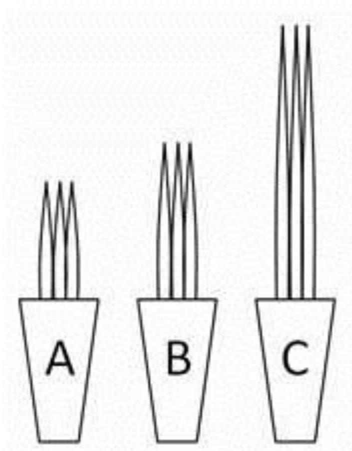
6. $\frac{99}{5} \; or \; \frac{99}{5} \times \frac{1}{2}$

7. $\frac{8}{21} \times 40 \; or \; \frac{28}{21} \times 40$

8. $\frac{8}{3} \times \frac{5}{7} \; or \; \frac{8}{3} \times \frac{9}{4}$

Grass Seedlings (5.NF.B.5)

The students in Raul's class were growing grass seedlings in different conditions for a science project. He noticed that Pablo's seedlings were $1\frac{1}{2}$ times a tall as his own seedlings. He also saw that Celina's seedlings were $\frac{3}{4}$ as tall as his own. Which of the seedlings shown below must belong to which student? Explain your reasoning.



Reasoning about Multiplication (5.NF.B.5)

Your classmate Ellen says,

When you multiply by a number, you will always get a bigger answer. Look, I can show you.

Start with 9.	
Multiply by 5.	$9 \times 5 = 45$
The answer is 45,	*and* $45 > 9$
45 is bigger than 9.	

It even works for fractions.	
Start with $\frac{1}{2}$.	
Multiply by 4.	$\frac{1}{2} \times 4 = 2$
The answer is 2,	*and* $2 > \frac{1}{2}$
2 is bigger than $\frac{1}{2}$.	

Ellen's calculations are correct, but her rule does not always work.

For what numbers will Ellen's rule work? For what numbers will Ellen's rule not work? Explain and give examples.

Running to School (5.NF.B.6)

The distance between Rosa's house and her school is $\frac{3}{4}$ mile. She ran $\frac{1}{3}$ of the way to school. How many miles did she run?

Drinking Juice (5.NF.B.6)

Alisa had $\frac{1}{2}$ a liter of juice in a bottle. She drank $\frac{3}{4}$ of the juice that was in the bottle. How many liters of juice did she drink?

Half a Recipe (5.NF.B.6)

Kendra is making $\frac{1}{2}$ of a recipe. The full recipe calls for $3\frac{1}{4}$ cup of flour. How many cups of flour should Kendra use?

Making Cookies (5.NF.B.6)

A recipe for chocolate chip cookies makes 4 dozen cookies and calls for the following ingredients:

- $1\frac{1}{2}$ C margarine
- $1\frac{3}{4}$ C sugar
- 2 tsp vanilla
- $3\frac{1}{4}$ C flour
- 1 tsp baking powder
- $\frac{1}{4}$ tsp salt
- 8 oz chocolate chips

1. How much of each ingredient is needed to make 3 recipes?
2. How much of each ingredient is needed to make $\frac{3}{4}$ of a recipe?

Dividing by One- Half (5.NF.B.7)

Solve the four problems below. Which of the following problems can be solved by finding $3 \div \frac{1}{2}$?

a. Shauna buys a three-foot-long sandwich for a party. She then cuts the sandwich into pieces, with each piece being $\frac{1}{2}$foot long. How many pieces does she get?

b. Phil makes 3 quarts of soup for dinner. His family eats half of the soup for dinner. How many quarts of soup does Phil's family eat for dinner?

c. A pirate finds three pounds of gold. In order to protect his riches, he hides the gold in two treasure chests, with an equal amount of gold in each chest. How many pounds of gold are in each chest?

d. Leo used half of a bag of flour to make bread. If he used 3 cups of flour, how many cups were in the bag to start?

How Many Servings of Oatmeal (5.NF.B.7)

A package contains 4 cups of oatmeal. There is $\frac{1}{3}$ cup of oatmeal in each serving.

How many servings of oatmeal are there in the package? Explain. Draw a picture to illustrate your solution.

Banana Pudding (5.NF.B.7)

Carolina's Banana Pudding Recipe
• 2 cups sour cream • 5 cups whipped cream • 3 cups vanilla pudding mix • 4 cups milk • 8 bananas

Carolina is making her special banana pudding recipe. She is looking for her cup measure, but can only find her quarter cup measure.

a. How many quarter cups does she need for the sour cream? Draw a picture to illustrate your solution, and write an equation that represents the situation.

b. How many quarter cups does she need for the milk? Draw a picture to illustrate your solution, and write an equation that represents the situation.

c. Carolina does not remember in what order she added the ingredients but the last ingredient added required 12 quarter cups. What was the last ingredient Carolina added to the pudding? Draw a picture to illustrate your solution, and write an equation that represents the situation.

Extensions and Sources

Online Resources

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Manipulatives

http://nlvm.usu.edu/en/nav/vlibrary.html
http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000
http://www.thinkingblocks.com/

Problem Solving Resources

*Illustrative Math Project**
http://illustrativemathematics.org/standards/k8
http://illustrativemathematics.org/standards/hs
The site contains sets of tasks that illustrate the expectations of various CCSS in grades K–8 grade and high school. More tasks will be appearing over the coming weeks. Eventually the sets of tasks will include elaborated teaching tasks with detailed information about using them for instructional purposes, rubrics, and student work.

***Inside Mathematics**
http://www.insidemathematics.org/index.php/tools-for-teachers
Inside Mathematics showcases multiple ways for educators to begin to transform their teaching practices. On this site, educators can find materials and tasks developed by grade level and content area.

IXL
http://www.ixl.com/

Sample Balance Math Tasks
http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

New York City Department of Education
http://schools.nyc.gov/Academics/CommonCoreLibrary/SeeStudentWork/default.htm
NYC educators and national experts developed Common Core-aligned tasks embedded in units of study to support schools in implementation of the CCSSM.

***Georgia Department of Education**
https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx
Georgia State Educator have created common core aligned units of study to support schools as they implement the Common Core State Standards.

Gates Foundations Tasks

http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12
http://www.misskoh.com

Math Score:
Math practices and assessments online developed by MIT graduates.
http://www.mathscore.com/

Massachusetts Comprehensive Assessment System
www.doe.mass.edu/mcas/search

Performance Assessment Links in Math (PALM)

PALM is currently being developed as an on-line, standards-based, resource bank of mathematics performance assessment tasks indexed via the National Council of Teachers of Mathematics (NCTM).
http://palm.sri.com/

Mathematics Vision Project
http://www.mathematicsvisionproject.org/

***NCTM**
http://illuminations.nctm.org/

Assessment Resources

- *Illustrative Math: http://illustrativemathematics.org/
- *PARCC: http://www.parcconline.org/samples/item-task-prototypes
- NJDOE: http://www.state.nj.us/education/modelcurriculum/math/ (username: model; password: curriculum)
- DANA: http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html
- New York: http://www.p12.nysed.gov/assessment/common-core-sample-questions/
- *Delaware: http://www.doe.k12.de.us/assessment/CCSS-comparison-docs.shtml

PARCC Prototyping Project		
Elementary Tasks (ctrl+click)	Middle Level Tasks (ctrl+click)	High School Tasks (ctrl+click)
• **Flower gardens (grade 3)** • **Fractions on the number line (grade 3)** • **Mariana's fractions (grade 3)**	• **Cake weighing (grade 6)** • **Gasoline consumption (grade 6)** • **Inches and centimeters (grade 6)**	• **Cellular growth** • **Golf balls in water** • **Isabella's credit card** • **Rabbit populations** • **Transforming graphs of**

• **School mural (grade 3)** • **Buses, vans, and cars (grade 4)** • **Deer in the park (grade 4)** • **Numbers of stadium seats (grade 4)** • **Ordering juice drinks (grade 4)**	• **Anne's family trip (grade 7)** • **School supplies (grade 7)** • **Spicy veggies (grade 7)** • **TV sales (grade 7)**	**quadratic functions**

Professional Development Resources

Edmodo

http://www.edmodo.com

Course: iibn34

Clark County School District Wiki Teacher

http://www.wiki-teacher.com/wikiDevelopment/unwrappedSearch.php#contentAreaId=6&courseId=474

Learner Express Modules for Teaching and Learning

http://www.learner.org/series/modules/express/videos/video_clips.html?type=1&subject=math

Additional Videos
http://www.achieve.org/achieving-common-core; http://www.youtube.com/user/TheHuntInstitute/videos

Mathematical Practices

Inside Mathematics

http://www.insidemathematics.org/index.php/commmon-core-math-intro

Also see the ***Tools for Educators***

The Teaching Channel

https://www.teachingchannel.org

***Learnzillion**

https://www.learnzillion.com

Engage NY

http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

**Adaptations of the these resources has been included in various lessons.*